

**LIFE ASSURANCE  
INVESTIGATION  
TABLES**

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Life Assurance Investigation Tables by Benjamin Hall Todd

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**BENJAMIN HALL TODD**

**LIFE ASSURANCE  
INVESTIGATION  
TABLES**



LIFE ASSURANCE  
INVESTIGATION TABLES,

SHOWING

THE VALUE OF £100 POLICY FOR ANY NUMBER OF YEARS,  
(NOT EXCEEDING FIFTY):

INTERPOLATED FOR MONTHS,

ACCORDING TO THE

CARLISLE TABLE OF MORTALITY  
AND 3 PER CENT. INTEREST;

ALSO,

ANNUAL AND SINGLE ASSURANCE PREMIUMS,  
CARLISLE TABLE—3 PER CENT. INTEREST,  
INTERPOLATED FOR MONTHS.

BY

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SECRETARY OF THE STANDARD LIFE ASSURANCE COMPANY.



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## INTRODUCTION.

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THE Valuation of Policies is one of the most important duties of the Actuary of a Life Assurance Company, as upon a proper estimate of the liabilities depends mainly the lasting prosperity and stability of such an institution.

The proper and most efficient method of estimating the liabilities of an Office is, the author believes, by the valuation of each Policy separately. A system of classifying Policies according to the age of the assured at the date of entry has, he is aware, been adopted; but such a method, while it shortens the operation, deprives the Actuary of those checks and periodical comparisons of results which are so important in such investigations.

With the aid of the Tables at present in use, the method of valuing Policies may be sufficiently curt and convenient for ordinary practical purposes; but when the Actuary has to investigate the affairs of a Company, and to ascertain its true position, the labour attending such an inquiry is very arduous; and from the immense mass of figures to be dealt with, all such investigations require to be very carefully checked to ensure that correctness which the importance of the subject demands—and, even then, errors are apt to creep in, notwithstanding the greatest vigilance of the computers. It was under these impressions that the idea of forming a Table of the Value of Policies first suggested itself to the author; and he now begs to submit his work, which he trusts will be found to be practically a useful one.

All recent observations, and the concurrent testimony of practical men, having tended to the adoption of the Carlisle Mortality Tables,

and 3 per cent. interest, as a sound basis for actual business transactions, the author has founded his calculations upon these data. The Carlisle Rate of Mortality, however, has been fixed upon for another reason, in addition to its having been proved by experience to be a correct Table, viz. :—Because Mr Sang's valuable work has, to a considerable extent, formed the groundwork of the Tables now presented.

#### TABLES.

TABLE I. shows the value of a Policy for £100, for all ages from 14 to 75 inclusive at date of entry, for every month, during a period of 50 years; but without the return of the proportion of Premium due to the assured, between successive payments, for the period from the date of surrender or valuation till the next Premium falls due.

This Table is intended for the daily use of Offices in estimating the surrender value of Policies, as well as for the purposes of investigation. The pure Premium returnable to the assured is not taken into account, to admit of the deduction of such per centage from the pure value as the Actuary may deem proper;—and in

TABLE II. will be found the pure Premium interpolated for months, to be added to the result given in Table I, *after* the deduction referred to has been made.

The Tables of the Value of Policies having been deduced from the values in Mr Sang's book, it may be necessary, at all events satisfactory, to give some account of the care bestowed by him to prevent errors; and the author cannot do better than quote, from the preface, Mr Sang's own words :—

“While offering to the public a work like the present, consisting principally of Tables, in the computation of which about two millions of figures have been written, it is right that some account be rendered of those precautions that have been taken for the purpose of guarding against error.

“Errors in a printed work, are either errors of the original manuscript, or errors of the compositor.

“To give a full account of the means that have been taken to prevent errors of the first kind, would be to discuss the whole plan and details of the computations, and would form almost a treatise on that subject; but an idea, sufficiently clear, may be obtained from a description of the precautions used in one or two of the branches of the calculation.

“The Tables of Short and Deferred Annuities were thus computed :—First, There were made two entirely separate sets of the

logarithms (to seven places) of the Short and of the Deferred Annuities; these sets were then carefully read over, and any differences which were found separately examined and again compared. From the logarithms of the Deferred Annuities, two sets of Deferred Annuities were taken out to two places beyond the printed results: these sets were then compared, and the two or three differences in the logarithms which had escaped notice were thus discovered. One set of Short Annuities was then obtained from the logarithms, and another set by subtracting the Deferred Annuities from the Annuity during the whole of life: and these two sets were read together, thus forming a closing check on the entire operation for Annuities.

"The computation of the Short and Deferred Assurances was conducted in the same manner; but that of the Policies and Risks was even more severely rigorous. In the first place, two sets of Policies, of logarithm Policies and of Risks, were computed by taking the difference between the values of the Assurance and of the remaining Premiums; comparisons being made at each step of the operation, but the sets being kept entirely distinct. After that, the logarithm Policies were computed from the differences of the Premiums, while the Risks were obtained by a totally independent and pretty intricate formula.

"In the other departments where a cross check could not readily be found, three separate sets of computations were made, and compared at each stage.

"By these means the chance of an error in the manuscript was rendered insignificant, almost zero: but an opportunity was again taken of comparing the different sets of results in the reading of the proofs; the first and second readings of the types being made from separate sets, and the final reading of the stereotypes being checked by four persons, the third set being used when it existed. The author, therefore, thinks himself warranted in expecting a degree of precision in these calculations not surpassed by any heavy tabular work yet printed."

Experience has gone far to realize Mr Sang's anticipations as to his work being free from error; and indeed it may be assumed with confidence to be correct.

Mr Sang has computed the Values of Policies on the assumption that the sum assured is to be payable *at the instant of death*, which is believed to have operated very materially against the work being so extensively used as it might otherwise have been.

The present Tables are computed as of £100 payable at the end of the year in which the life may fail,—or, in other words, payable six months after death,—which, on account of the Tables contained in the numerous works already published being framed on that understanding, and the general practice of Offices, will render them practically useful.

The following are the logarithms of the Values of £1 payable



at the instant of death, and £1 payable at the end of the year in which the life shall fail :—

Age.	Log. of Value of £1, payable at Instant of Death— Sang.	Log. of Value of £1, payable at end of year in which Life shall fail.*	Difference.†
15	1.5021876	1.4957474	.0064202
25	.6733822	.6669621	.0064201
35	.6438769	.6374608	.0064161
45	.7180081	.7065871	.0064160
55	.7918908	.7849419	.0064189
65	.8588748	.8519688	.0064165
75	.9160681	.9086020	.0064206
85	.9498296	.9429098	.0064168
95	.9660868	.9496703	.0064165

\* These Values are calculated from Mr Thomson's "Carlisle 3 per Cent." Table, and agree with the Values given by Messrs Gray, Smith, and Orchard, in their "Assurance and Annuity Tables," recently published.

† The true difference, as shown hereafter, is .0064186, and the above variation is in consequence of Mr Sang's work only giving the Values to 5 decimal places.

From the preceding Table, it appears that the difference is Log. .00642, nearly, which is found in the following manner :—

*Interest of £1 for one year, 3 per cent., + unity = 1.03.*

Let  $x$  = present value of £1, payable at end of year in which a life fails (that is, 6 months after death)—

Then  $\frac{x}{1.03}$  = present value of £1, payable 6 months after death, *deferred one year.*

And  $\frac{x}{(1.03)^2}$  = present value of £1, payable 6 months after death, *deferred two years.*

Again—

$x(1.03)^{\frac{1}{4}}$  = present value of £1 payable 6 months after death, *anticipated a quarter of a year,—or, payable three months after death—*

Then  $\text{Log. } \{x(1.03)^{\frac{1}{4}}\} = \text{Log. } x + \frac{1}{4} \text{Log. } 1.03 = \text{Log. } x + \frac{.0128372}{4}$

Therefore  $\text{Log. } \{x(1.03)^{\frac{1}{4}}\} = \text{Log. } x + .0032093$

Or  $\text{Log. } x = \text{Log. } \{x(1.03)^{\frac{1}{4}}\} - .0032093$

Also  $x(1.03)^{\frac{1}{2}}$  = present value of £1 payable 6 months after death, *anticipated half a year,—or, payable at instant of death—*

Then  $\text{Log. } \{x(1.03)^{\frac{1}{2}}\} = \text{Log. } x + \frac{1}{2} \text{Log. } 1.03 = \text{Log. } x + \frac{.0128372}{2}$

Therefore  $\text{Log. } \{x(1.03)^{\frac{1}{2}}\} = \text{Log. } x + .0064186$

Or  $\text{Log. } x = \text{Log. } \{x(1.03)^{\frac{1}{2}}\} - .0064186$

So that, having the logarithm of the Value of £1 payable at instant of death, .0032093 has to be deducted in order to postpone the payment of the sum *three months*; and .0064186 has to be deducted to make the sum payable *six months after death*, as given in Table I. of this work. And, consequently, .0032093 has to be added to the log. of the Values in Table I. to obtain the log. of the Value of the sum payable at three months after death.

The following, then, is the course which has been pursued in the computation of the Tables of this work, and the means employed to prevent errors: .00642 was deducted from the logarithms of the Values in Mr Sang's Table, and the result put down. This result was then read back, with the addition of .00642, to Mr Sang's Table, to form a check. The numbers were then found by a table of anti-logarithms, and these numbers checked by a different work of logarithms. The interpolations were then made, which, in the operation, form a complete check on the computation at the end of each line. The manuscript was then set up in type; the printed proof was then checked by a separate interpolation, thus forming a check on the manuscript and setting; the errors were then examined and rectified; and, lastly, the stereotype proof was compared with the original corrected type proof. With these precautions, the author trusts that the Tables will be found to be as free from error as any equal amount of tabular work yet published.

### EXPLANATION OF THE TABLES.

TABLE I. shows the value of a Policy for £100, for all ages from 14 to 75 inclusive at date of entry, for every month, during a period of 50 years; but the columns headed " $\frac{1}{12}$ " to " $\frac{11}{12}$ ," which contain the values at the end of each month during the year, do not include the proportion of Premium returnable to the Assured, for risk not incurred, in consequence of the Policy being surrendered before the expiry of the period for which the Premium has been paid.

TABLE II. shows the Premium returnable for the portion of the year unexpired, for every month, for all ages from 14 to 75 inclusive, required to complete the values in Table I. contained in the columns headed " $\frac{1}{12}$ " to " $\frac{11}{12}$ ."

In explanation of the arrangement of the values in Tables I. and II., take, for example, Table I. "10 years." The column "0" contains the values of Policies which have been 10 completed years in force, or on which 10 Premiums have been paid, the 11th Premium being just due but not paid. The other columns headed " $\frac{1}{12}$ " to " $\frac{11}{12}$ ," contain the values of Policies which have been in existence for 10 years and 1 month to 10 years and 11 months, on which consequently the 11th Premium has been paid; and to complete the value of the Policy, the proportion of Premium corresponding to the age at entry, and the number of months from the date of valuation till the next Premium falls due, given in Table II., has to be added, as shown in the following examples.

The "1st Age" column contains the age of the life assured at *date of entry*. The "2nd Age" column contains the *age attained* at date of valuation, which facilitates the reference to the Table, and gives the computer a ready check that he is extracting the correct value.

TABLE III. shows the present value of £1 payable at the end of the year in which an assigned life may fail, for every month of age, from 14 to 104 inclusive, Carlisle 3 per cent., to be used in the valuation of Bonus Additions to Policies, or of Policies of which the Premiums have been compensated by single payment or otherwise.