GEOMETRICAL PROBLEMS IN THE PROPERTIES OF THE CONIC SECTIONS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649320769

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Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

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H. LATHAM

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GEOMETRICAL PROBLEMS

IN THE PROPERTIES OF THE CONIC SECTIONS.

BY H. LATHAM, B.A.

TUTOR OF TRINITY HALL, CAMBRIDGE.

CAMBRIDGE:
MACMILLAN, BARCLAY, AND MACMILLAN;
LONDON: G. BELL, FLEET STREET.

1848

CAMBRIDGE
PRINTED BY METCALFE AND PALMER, TRINITY-STREET.

PREFACE.

Since, by the recent alterations in the Senate-House Examination, Candidates for Honors are required to be able to prove the properties of the Conic Sections by Geometrical methods, it appeared to the Author that a Collection of Problems soluble by such means would be found useful.

Every question in the following pages has been worked out geometrically, but some theorems have been introduced, in which the result is put under an ungeometrical form, either as being more familiar or more symmetrical (*e. g.* Prob. 66, *Ellipse*, and Prob. 94, *Parabola*): in these it is the last step only which is not geometrical.

Answers have been always given, and Solutions in cases where the property was so important as to be necessary for a thorough knowledge of the subject, or where any useful method of proceeding could be exemplified.

Some propositions have also been proved as being of extensive application in the solution of Problems in Central Forces, without the Differential Calculus, because it was the Author's intention to accompany these pages by a series of illustrations of the three first Sections of Newton, which he has since resolved on publishing separately.

Many of the problems have been taken from the works of Emerson, Wallace, and other geometrical writers on the subject, as well as from various Cambridge and Dublin Examination Papers. Such as were suitable in the Senate-House Papers of 1848 were inserted, with their solutions, while the work was in the press, excepting one in the paper of problems on early subjects, which is included in Prob. 31, *Ellipse:* with the exception of these, and a few others added at the same time, the questions have been as much as possible arranged progressively.

Trinity Hall, February 1848.

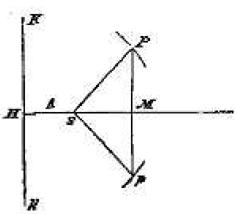
CONIC SECTIONS.

THE PARABOLA.

WHEREVER, in what follows, a parabola is required to be described, fulfilling certain conditions, the problem may be considered to be solved when the focus and directrix are found, as then any number of points may be found in the curve, as will be shewn.

1. Given the focus and directrix; to determine any number of points in the curve.

Let S be the focus and KHR the directrix. From S draw SH perpendicular to KR, and produce it indefinitely in direction HS. Bisect SH in A, take any point M in this line, not between A and H; through M draw an indefinite straight line PMp perpendicular to HM, with centre S and radius HM, describe a circle cutting PMp in P and p; P and p will be points in the parabola. For

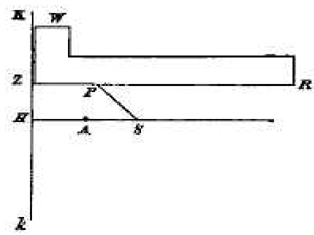


$$SP = Sp = HM = HA + AM = AS + AM,$$

the property of a parabola where the focus is S and vertex A.

2. Given the focus and directrix; shew how the parabola may be described practically.

Let WZR be a ruler, having a right angle at Z, and let the end ZW slide along the line KHk. Let one end of a string, whose length is RZ, be fixed at R and the other k at the focus S. Then if P be a moveable pencil, always so applied as to keep the string stretched, it will trace out a parabola.

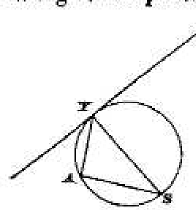


For $SP + PR$ = length of string = $ZR = ZP + PR$, therefore $SP = ZP$; the property of a parabola whose focus is S and whose directrix is KHk.

The above is a simple case of finding a geometrical *locus*. In finding the locus of a point moving according to a fixed law, we must endeavour to find some distinguishing property of a known curve (in what follows a circle or a Conic Section) which is possessed by the point in all its positions. This curve is said to be the *locus* of the point.

3. Find the locus of the vertices of all the parabolas which have a given focus and touch a given straight line.

Let S be the focus, YZ the straight line given in position. Draw SY perpendicular upon YZ; then the tangent at the vertex A of any parabola, to which YZ is a tangent, must pass through Y; also SA must be perpendicular to it. Hence that A may be the vertex of one of the required parabolas, it must be the vertex of a right-angled triangle described upon SY; all which vertices lie in the circle described on SY. This circle is therefore the locus of the vertices of the parabolas described as required.



Sometimes when the position of a point is required, which has to fulfil two conditions, the locus corresponding to each condition may be constructed, and the point or points are determined by the intersection of the loci. This method and the result of problem (3) are useful in what follows.

4. Describe a parabola with a given focus and latus rectum, touching a given straight line.

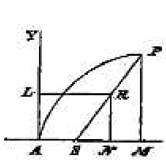
5. Given the focus and position of the axis; describe a parabola touching a straight line given in position.

6. Describe a parabola with a given focus touching two straight lines given in position.

7. Given a straight line and two points on the same side of it; shew that in general two parabolas may be described passing through the points and having the line for directrix. Also shew that when only one parabola can be described, the distance of the points from each other equals the sum of their distances from the directrix.

8. A parabola being traced and its axis given, determine the focus. A method nearly the inverse of that used in Prob. 1, may be used here.

9. The circle described upon the radius vector SP of a parabola touches the tangent at the vertex.



Let SP be the radius vector, Y the tangent at the vertex, bisect SP in R and draw LR perpendicular to AY. Drop perpendiculars PM, RN, on the axis; then, because $SP = 2SR$, by similar triangles SRN, SPN,

$$SM = 2SN,$$

and
$$RL = AN = AS + SN = \frac{AS}{2} + \frac{AS}{2} + \frac{SM}{2}$$

$$= \frac{AS + AM}{2} = \frac{SP}{2},$$

therefore $RL = RS = RP$, and the circle described upon SP, with centre R, will touch AY in L. Q. E. D.