

**AN ELEMENTARY TREATISE  
ON THE THEORY OF  
DETERMINANTS: A TEXT-  
BOOK FOR COLLEGES**

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An Elementary Treatise on the Theory of Determinants: A Text-Book for Colleges by Paul H. Hanus

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**PAUL H. HANUS**

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Wm. J. Bussey  
J. C. ...  
Nov. 26, 1886.

AN

ELEMENTARY TREATISE

ON THE

THEORY OF DETERMINANTS.

A TEXT-BOOK FOR COLLEGES.

BY

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FORMERLY PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF COLORADO;  
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No. 2 (WEST DENVER).

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## PREFACE.

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THE importance of a knowledge of DETERMINANTS to all who extend their reading beyond the elements of mathematics, and the fact that most modern writers employ the determinant notation, have led to the belief that an American work on Determinants might satisfy a growing demand.

This is a text-book, and not an exhaustive treatise. Enough is given, however, to enable the student to use the determinant notation with ease, and to enable him to pursue his further reading in the modern higher mathematics with pleasure and profit.

The book is written with reference to the wants of the private student as well as to the needs of the class-room. The subject is at first presented with great simplicity. As the student advances, less attention is given to details. More than half the volume is devoted to applications and special forms, that the reader may get *some* notion of the power and utility of determinants as instruments of research.

Throughout the work care has been taken to show how each new concept has been evolved naturally; and, whenever it is thought advisable, a special case precedes the general discussion.

The work has been written in the far West, where contact with others in the same field was practically impossible. I

shall therefore be grateful for any notification of errors that may have escaped detection.

My thanks are due to Messrs. J. S. CUSHING & Co., of Boston, for great care and patience manifested in the preparation of the plates.

Among the works consulted most assistance has been derived from the following. All the works named have been used freely.

**Matska.** — Grundzüge der systematischen Einführung und Begründung der Lehre der Determinanten.

**Baltzer.** — Theorie und Anwendung der Determinanten (Fünfte Auflage).

**Günther.** — Lehrbuch der Determinanten-Theorie (Zweite Auflage).

**Diekmann.** — Einleitung in die Lehre von den Determinanten und ihrer Anwendung auf, etc.

**Dostor.** — Éléments de la Théorie des Déterminants avec Applications, etc. (Deuxième édition).

**Höfel.** — Cours de Calcul Infinitésimal.

**Scott.** — A Treatise on the Theory of Determinants and their Applications, etc.

**Burnside and Panton.** — The Theory of Equations, with an Introduction, etc.

**Muir.** — A Treatise on the Theory of Determinants.

I am especially indebted to the last two works for many examples.

PAUL H. HANUS.

BOULDER, COL., May, 1886.



# CONTENTS.

## CHAPTER I.

### PRELIMINARY NOTIONS AND DEFINITIONS.

ART.	PAGE.
1. Discovery of Determinants . . . . .	1
2-7. Determinants produced by eliminating the unknowns from a system of simultaneous equations . . . . .	2-8
8-10. Values of the unknowns in determinant form . . . . .	8-10
9. Change of sign . . . . .	9
11. Notation . . . . .	11-12
12-14. Expansions with square notation . . . . .	13
15. Rule for expanding a determinat of the third order . . . . .	14
Examples . . . . .	14-16

## CHAPTER II.

### GENERAL PROPERTIES OF DETERMINANTS.

16-19. Definition and notation . . . . .	17-20
20. Corollaries . . . . .	20
21. Inversions of order . . . . .	20-21
22. Number of terms in a determinant . . . . .	21
23-24. Corollaries; expansions . . . . .	22-23
25. If the rows in order are made the columns in order, etc. . . . .	23
26. Number of positive and negative terms . . . . .	24
27. Interchange of two parallel lines . . . . .	24

ART.	PAGE.
28. Two identical parallel lines . . . . .	25
29. Cyclical permutations . . . . .	25-26
30. Corollary . . . . .	26
Examples . . . . .	27
31. Every element of a line multiplied by the same number . . . . .	27
32-33. Corollaries . . . . .	28
34-35. Decomposition of determinant with polynomial ele- ments . . . . .	28-29
36. Converse of 34 . . . . .	30
37. Transformation by addition of parallel lines . . . . .	30-31
38. Minor determinants, or Minors . . . . .	31-32
39. Expansion of determinant as linear function of the elements of one line . . . . .	32
40. Coefficient of any element in the expansion of a determinant . . . . .	33-34
41-44. Corollaries; expansions . . . . .	34-36
Examples . . . . .	36-40
45. Elements of a line multiplied by first minors of corre- sponding elements of a parallel line . . . . .	40-41
47. Expansion in zero-axial determinants . . . . .	41-42
48-49. Simplification by taking each consecutive pair of ele- ments in the first row, etc. . . . .	43-44
Examples . . . . .	45-46
50-54. The product of two determinants . . . . .	46-53
Examples . . . . .	53-56
55. Laplace's Theorem (expansion) . . . . .	56-57
57. Product of a determinant by one of its minors . . . . .	58-60
58. Rectangular Arrays or Matrices (product of) . . . . .	61-63
59-62. The Reciprocal or adjugate determinant . . . . .	63-68
Examples . . . . .	69-71
63. Special expansions (including Cauchy's Theorem, Example III.) . . . . .	71-76

CONTENTS.

vii

ART.		PAGE.
64.	Solutions of certain determinant equations . . . . .	76-78
65-66.	Differential of a determinaat . . . . .	79-81

CHAPTER III.

APPLICATIONS AND SPECIAL FORMS.

67-70.	Solution of a system of simple simultaneous equations . . . . .	82-86
71-72.	If the equations of the system are not independent . . . . .	86
73.	If $m_1 = m_2 = \dots = m_{n-1} = 0$ , and one $m$ as $m_n$ does not . . . . .	87
74-78.	Solution of a system of linear homogeneous equations . . . . .	87-92
77.	The condition $\Delta = 0$ for a system of homogeneous equations . . . . .	91
79.	Condition fulfilled when $n$ equations containing $n-1$ unknowns are simultaneous . . . . .	92-93
80.	Solution of a system of simple equations by 79 . . . . .	93-94
81.	Another application of 79 . . . . .	94
82-83.	The Matrix as the result of elimination . . . . .	95-97
84-89.	Applications of preceding processes . . . . .	98-110
90-100.	Resultants, or Eliminants . . . . .	110-126
91.	Euler's Method of elimination . . . . .	111-112
92.	Sylvester's Method . . . . .	113-114
93.	Bezout's Method (Cauchy) . . . . .	114-118
94-95.	Resultant in terms of the roots . . . . .	118-121
96.	Properties of Resultant . . . . .	121-122
97-100.	Applications of Sylvester's Method . . . . .	122-126
101-102.	Discriminant of an equation . . . . .	126-129
103.	Resultant of a system of homogeneous equations when $n-1$ are linear, and one is of the second degree . . . . .	129-130