PRINCIPLES OF MODERN GEOMETRY, WITH NUMERIOUS APPLICATIONS TO PLANE AND SPHERICAL FIGURES; AND AN APPENDIX, CONTAINING QUESTIONS FOR EXERCISE. INTENDED CHIEFLY FOR THE USE OF JUNIOR STUDENTS

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Principles of modern geometry, with numerious applications to plane and spherical figures; and an appendix, containing questions for exercise. Intended chiefly for the use of junior students by John Mulcahy

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JOHN MULCAHY

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PREFACE.

THE object of these pages is to lay down and illustrate the more elementary principles of those Geometrical Methods which, in recent times, have been so successfully employed to investigate the properties of figured space.

The importance of the principles in question seems to render it advisable that the student should enter on their application at an early period of his progress; and, in accordance with this view, examples in Plane and Spherical Geometry are here given in considerable numbers.

The scope and extent of the present work may be collected with tolerable accuracy from the Table of Contents; but it is necessary to state, for the information of the reader, the amount of Mathematical knowledge which he is supposed to possess. The preliminary Propositions required for the perusal of the first five Chapters are to be found, with few exceptions, in the first six Books of Euclid's Elements. Some occasional deductions, involving the formulæ of Plane Trigonometry, are appended to these Chapters in the form of Notes. In the sixth Chapter the fundamental notions of Algebraic Geometry are referred to. The seventh, eighth, ninth, and tenth Chapters presuppose an acquaintance with the ordinary principles of Spherical Trigonometry; and in the last two Chapters some of the properties of Curves of the second degree are assumed.

From this statement it will be gathered, that the work is in a great degree of a *supplementary* nature, and that the subjects embraced have some diversity of character. It has been attempted, however, to preserve throughout a certain unity of design, and a due connexion of the various parts.

Those who are acquainted with the writings of Poncelet and Chasles will readily appreciate the extent to which the author has borrowed from these distinguished geometers in the present publication. He is also indebted to the Additions contained in Professor Graves's Translation of Chasles's Memoirs of Cones and Spherical Conics; and on several occasions he has consulted with advantage Dr. Salmon's Treatise on Conic Sections. It is to be added, that many of the examples throughout the work, and of the questions in the Appendix, are taken from the Examination Papers of Trinity College, Dublin.

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PRINCIPLES OF MODERN GEOMETRY.

CHAPTER I.

HARMONIC PROPORTION AND HARMONIC PENCILS.

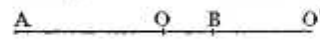
ART. 1. THREE quantities are said to be in harmonic proportion, when the first is to the third as the difference between the first and second is to the difference between the second and third. Thus, 3, 4, 6, are in harmonic proportion.

It follows from this definition, that the three quantities will still be in harmonic proportion when their order is inverted, or when they are altered in the same ratio. Thus 6, 4, 3, are in harmonic proportion, and so also are $3m$, $4m$, $6m$, m being any number.

Three quantities are said to be in arithmetic proportion, when the difference of the first and second is equal to the difference of the second and third.

Three quantities are said to be in geometric proportion, when the first is to the second as the second to the third; that is, when they are proportional in the sense of the Fifth Book of Euclid.

The relation between these three kinds of proportion may be thus exhibited :—

In arithmetic proportion the differences are equal; that is, they are as the *first* to *itself*. In geometric proportion they are as the first to the *second* (by conversion and alternation). And in harmonic proportion they are as the first to the *third*.

2. Let a right line AB, be cut internally at O, and externally at O′ in the same ratio; that is, so that AO : BO :: AO′ : BO′; then,