

# **THE ELEMENTS OF SOLID GEOMETRY**

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The Elements of Solid Geometry by William C. Bartol

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**WILLIAM C. BARTOL**

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SOLID GEOMETRY**



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OF  
SOLID GEOMETRY.

BY  
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## PREFACE.

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I HAVE written this book, having in view the ultimate improvement of the course in mathematics offered at Bucknell University, and bearing in mind that the crowded curricula give to this course less time and more subjects than was the case twenty-five years ago.

In carrying forward a course of mathematical study, nothing can make amends for hasty or imperfect preparation. However, since Plane Geometry is an almost universal requirement for admission to college, it becomes possible, by means of the entrance examinations, to enter in the subject of Solid Geometry, only those students who are already well trained in Euclidian methods of demonstration and investigation.

Believing that for such students the course in Solid Geometry may be made quite brief with the ultimate advantage of having more time for advanced mathematics, I offer this short course. In it are a number of theorems for original demonstration and many illustrative examples. A section on Mensuration is introduced with the design of calling special attention, by means of illustrative examples, to all the important rules for finding volumes and surfaces of solids, demonstrated in the preceding sections. Also, methods for finding the volumes of the Regular Polyhedron, the Wedge, and the Prismoid are deduced.

For the purpose of bringing the important theorems as near as possible to the definitions, postulates, etc., on which they rest, I have found it necessary to deviate somewhat from the usual sequence of propositions. Thus, I have grouped in the same section the prism and its limiting case, the cylinder, because they have so many properties in common. I have treated the pyramid and its limiting case, the cone, in like manner, etc.

Always, I have aimed to give the most direct proof possible, and to save the student, by means of corollaries, the labor of reproducing constructions unnecessarily.

An experience of twenty years in teaching mathematics leads me to think that the student who gets up the subject from this brief work, in the end will be at no disadvantage from not having used some one of our larger popular text-books.

Many of the diagrams used in illustration are, by permission, from Professor Wells' geometry. In thanking him for this act of courtesy, I desire also to acknowledge my indebtedness to him for valuable aid rendered me through the agency of his text-books, some of which I have had in class-room use from the date of their publication.

WILLIAM C. BARTOL.

BUCKNELL UNIVERSITY,  
LEWISBURG, PA., Aug., 1883.



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## THE ELEMENTS OF SOLID GEOMETRY.

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### SECTION I.

#### LINES AND PLANES IN SPACE

##### DEFINITIONS.

**1.** A *plane* is a surface such that, if a straight line be passed through any two of its points, the line will lie wholly in the surface.

When a line lies wholly in a plane, it may be said of the plane that it passes through the line.

**2.** The *intersection* of two surfaces is a line containing all the points which are common to the two surfaces.

**3.** The *intersection* of a line and a plane is the point where the line pierces the plane. This point is called the *foot* of the line.

**4.** A line is *perpendicular* to a plane when it is perpendicular to every line of the plane, passing through its foot. And the plane is then perpendicular to the line.

**5.** Planes are *parallel* if they do not meet, however far extended.

**6.** A line and a plane are *parallel* if they do not meet, however far extended.