

ARITHMETIC: RULES AND REASONS

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Arithmetic: Rules and Reasons by John H. Boardman

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JOHN H. BOARDMAN

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AND REASONS**

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RULES AND REASONS.

BY

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P R E F A C E.

THE teaching of the higher branches of Arithmetic forming part of my duties, as Mathematical Master of the Free Grammar School, Manchester, I could find no treatise which I considered suited to the wants of a school composed of boys intended for commercial pursuits, or for some profession; for though it was of importance to teach principles to all, the former required knowledge in certain commercial rules which it has of late been the custom to defer to a point which, although the place where they could best be discussed, was too far advanced for the boys who might be removed early from school, and never properly comprehended by others.

Whilst I admit the general principles on which they depend must always be there considered, still, at least in detail, all may be understood before; and, as the simple questions upon them would form better exercises than more numerous examples in the earlier rules, I deem the advantage in inserting them before Fractions will be easily perceived by those practically engaged in education. I have, however, endeavoured to make clear, when Fractions have been discussed, the general principles of all such cases, and have added more complicated questions, and other cases not to be understood before, or of much less importance. But I would recommend the study of the principles as soon as the intellect of the boy may be sufficiently matured. There can be no advantage

in the knowledge of many rules, or mere accuracy of calculation, when the reason of the process is unknown;—the understanding must be enlisted before the results obtained are acted upon in the affairs of daily life.

The merchant who, in the course of his business, performs the most minute calculations, I venture to say owes his knowledge far less to the rules he has learnt at school, than to his own mental powers having taught him the laws which apply to the cases he is ordinarily engaged in;—the subject is, in fact, too important to allow him to be the blind follower of a rule.

I have endeavoured to shew the reason of every rule I have given, but, as I conceive to mingle rules and reasons would only tend to confuse when the latter were beyond the power of the student, whilst I have given such explanations as I consider ought never to be omitted, I have kept the more difficult proofs distinctly separate, under every rule, but at the bottom of each page; the proof of the example preceding, in most cases, that of the rule.

The tutor will discover and direct the omission of proofs which, from the nature of the case, are too difficult of comprehension; and as I have, by my mode of printing, made this easily practicable, I may be excused in having given several entirely omitted in any Arithmetic which has come under my notice.

A collection of examples will probably follow; in the meantime I recommend "Thrower's," which appears, in most respects, well arranged.

October 15th, 1850.

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TABLES.

| ADDITION. | | | | | | | | SUBTRACTION. | | | | | | | |
|-----------|----|----|----|----|----|----|----|--------------|----|----|----|----|----|----|----|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

MULTIPLICATION AND DIVISION.

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|----|-----|----|-----|----|-----|
| 1 | 2 | 1 | 3 | 1 | 4 | 1 | 5 | 1 | 6 | 1 | 7 | 1 | 8 | 1 | 9 | 1 | 10 | 1 | 11 | 1 | 12 |
| 2 | 4 | 2 | 6 | 2 | 8 | 2 | 10 | 2 | 12 | 2 | 14 | 2 | 16 | 2 | 18 | 2 | 20 | 2 | 22 | 2 | 24 |
| 3 | 6 | 3 | 9 | 3 | 12 | 3 | 15 | 3 | 18 | 3 | 21 | 3 | 24 | 3 | 27 | 3 | 30 | 3 | 33 | 3 | 36 |
| 4 | 8 | 4 | 12 | 4 | 16 | 4 | 20 | 4 | 24 | 4 | 28 | 4 | 32 | 4 | 36 | 4 | 40 | 4 | 44 | 4 | 48 |
| 5 | 10 | 5 | 15 | 5 | 20 | 5 | 25 | 5 | 30 | 5 | 35 | 5 | 40 | 5 | 45 | 5 | 50 | 5 | 55 | 5 | 60 |
| 6 | 12 | 6 | 18 | 6 | 24 | 6 | 30 | 6 | 36 | 6 | 42 | 6 | 48 | 6 | 54 | 6 | 60 | 6 | 66 | 6 | 72 |
| 7 | 14 | 7 | 21 | 7 | 28 | 7 | 35 | 7 | 42 | 7 | 49 | 7 | 56 | 7 | 63 | 7 | 70 | 7 | 77 | 7 | 84 |
| 8 | 16 | 8 | 24 | 8 | 32 | 8 | 40 | 8 | 48 | 8 | 56 | 8 | 64 | 8 | 72 | 8 | 80 | 8 | 88 | 8 | 96 |
| 9 | 18 | 9 | 27 | 9 | 36 | 9 | 45 | 9 | 54 | 9 | 63 | 9 | 72 | 9 | 81 | 9 | 90 | 9 | 99 | 9 | 108 |
| 10 | 20 | 10 | 30 | 10 | 40 | 10 | 50 | 10 | 60 | 10 | 70 | 10 | 80 | 10 | 90 | 10 | 100 | 10 | 110 | 10 | 120 |
| 11 | 22 | 11 | 33 | 11 | 44 | 11 | 55 | 11 | 66 | 11 | 77 | 11 | 88 | 11 | 99 | 11 | 110 | 11 | 121 | 11 | 132 |
| 12 | 24 | 12 | 36 | 12 | 48 | 12 | 60 | 12 | 72 | 12 | 84 | 12 | 96 | 12 | 108 | 12 | 120 | 12 | 132 | 12 | 144 |

For Multiplication, say "twice one are two, twice two are four," &c.; "three times one are three," &c.

For Division, say "twos in two are one, twos in four are two," &c.; "threes in three are one," &c.

MONEY TABLES.

MONEY.

| | | | | | | | | | |
|---------------|-----|-------------|-----------------------------|-----|---|-------------------------|--|--|--|
| | | | | | | | | | |
| | 2 | farthings | (<i>f.</i> or <i>gr.</i>) | are | 1 | halfpenny. | | | |
| | 2 | halfpennies | | " | 1 | penny (<i>d.</i>) | | | |
| | 4 | pennies | | " | 4 | pence. | | | |
| | 6 | pennies | | " | 6 | pence. | | | |
| | 12 | pence | | " | 1 | shilling (<i>s.</i>) | | | |
| 2s. 6d. or 30 | | pence | | " | 1 | half-crown. | | | |
| 5 0 " | 60 | pence | | " | 1 | crown. | | | |
| 10 0 | ... | ... | | " | 1 | half-sovereign. | | | |
| 20 0 | ... | ... | | " | 1 | sovereign or pound (£). | | | |

| FARTHINGS. | | | PENCE. | | | SHILLINGS. | | |
|-------------|--------------|-----------|-----------|-----------|-----------|------------|-----|-----------|
| <i>far.</i> | <i>half.</i> | <i>d.</i> | <i>d.</i> | <i>s.</i> | <i>d.</i> | <i>s.</i> | £ | <i>s.</i> |
| 2 | are | 1 | 12 | are | 1 0 | 20 | are | 1 0 |
| 4 | or | 2 | 18 | " | 1 6 | 30 | " | 1 10 |
| 6 | " | 3 | 20 | " | 1 8 | 40 | " | 2 0 |
| 8 | " | 4 | 24 | " | 2 0 | 50 | " | 2 10 |
| 10 | " | 5 | 30 | " | 2 6 | 60 | " | 3 0 |
| 12 | " | 6 | 36 | " | 3 0 | 70 | " | 3 10 |
| 14 | " | 7 | 40 | " | 3 4 | 80 | " | 4 0 |
| 16 | " | 8 | 42 | " | 3 6 | 90 | " | 4 10 |
| 18 | " | 9 | 48 | " | 4 0 | 100 | " | 5 0 |
| 20 | " | 10 | 50 | " | 4 2 | 110 | " | 5 10 |
| 22 | " | 11 | 54 | " | 4 6 | 120 | " | 6 0 |
| 24 | " | 12 | 60 | " | 5 0 | 130 | " | 6 10 |
| 26 | " | 13 | 66 | " | 5 6 | 140 | " | 7 0 |
| 28 | " | 14 | 70 | " | 5 10 | 150 | " | 7 10 |
| 30 | " | 15 | 72 | " | 6 0 | 160 | " | 8 0 |
| 32 | " | 16 | 78 | " | 6 6 | 170 | " | 8 10 |
| 34 | " | 17 | 80 | " | 6 8 | 180 | " | 9 0 |
| 36 | " | 18 | 84 | " | 7 0 | 190 | " | 9 10 |
| 38 | " | 19 | 90 | " | 7 6 | 200 | " | 10 0 |
| 40 | " | 20 | 96 | " | 8 0 | 210 | " | 10 10 |
| 42 | " | 21 | 100 | " | 8 4 | 220 | " | 11 0 |
| 44 | " | 22 | 102 | " | 8 6 | 230 | " | 11 10 |
| 46 | " | 23 | 108 | " | 9 0 | 240 | " | 12 0 |
| 48 | " | 24 | 110 | " | 9 2 | 250 | " | 12 10 |
| 50 | " | 25 | 114 | " | 9 6 | 260 | " | 13 0 |
| 60 | " | 30 | 120 | " | 10 0 | 270 | " | 13 10 |
| 70 | " | 35 | 126 | " | 10 6 | 280 | " | 14 0 |
| 72 | " | 36 | 130 | " | 10 10 | 290 | " | 14 10 |
| 80 | " | 40 | 132 | " | 11 0 | 300 | " | 15 0 |
| 90 | " | 45 | 138 | " | 11 6 | 400 | " | 20 0 |
| 96 | " | 48 | 140 | " | 11 8 | 500 | " | 25 0 |
| 100 | " | 50 | 144 | " | 12 0 | 1000 | " | 50 0 |