

**THE THEORY OF THE  
RELATIVITY  
OF MOTION**

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The theory of the relativity of motion by Richard C. Tolman

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**RICHARD C. TOLMAN**

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Physics  
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THE THEORY OF  
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BY  
RICHARD C. TOLMAN

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# THE THEORY OF THE RELATIVITY OF MOTION.

BY

RICHARD C. TOLMAN, PH.D.

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