THEORY OF THE MOON'S MOTION

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BY

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PREFACE.

An abstract of the following investigation was published in Nos. (2024–2026) of the *Astronomische Nachrichten*. But, a few verbal inaccuracies, together with numerous typographical errors, being found in the work as printed, it was decided to reprint the whole investigation, giving the mathematical developments more in detail, and also developing the formula for the latitude of the moon, a subject not touched upon in that investigation. The author hopes to find leisure for the complete development of the perturbations of the moon's motions, by means of the differential equations given in this first chapter. It is believed that these equations will give the co-ordinates of the moon in a more direct and simple manner than any combination of the general differential equations of motion hitherto employed for that purpose.

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INTRODUCTION.

1. It is now nearly two centuries since the theory of the universal gravitation of matter according to the law of the inverse square of the distances was discovered and subjected to calculation by NEWTON. The theory when regarded as a law of nature may be stated as follows: *Every particle of matter in the universe attracts every other particle with a force which varies directly as its mass, and inversely as the square of the distance between them.*

Mathematicians have attempted to deduce from this law of matter all the phenomena attending the motions of the heavenly bodies. For this purpose the sun, planets and satellites may be regarded as particles of matter when compared with the distances which separate them from the other bodies of the system, the masses of the planets being regarded as infinitely small in comparison with that of the sun. The principal mathematical consequences resulting from the operation of this law of matter may be stated as in the three following theorems:

I. *The orbits of the planets and comets are conic sections in which the sun occupies the principal focus;*

II. *The radius vector of each planet or comet sweeps over equal areas in equal times;* and

III. *The squares of the times of revolution of the different planets are to each other in the same proportion as the cubes of their mean distances from the sun.*

These three laws, which were discovered by KEPLER, may be regarded as the embodiment of the theory of universal gravitation, and are the foundation of physical or mathematical astronomy. They are, however, exact only on the supposition that the masses of the planets are infinitely small in comparison with that of the sun, and hence are not applicable to the solar system without some modification, because the masses of some of the planets are finite instead of being infinitely small. The motions of the planets therefore do not strictly conform to the preceding laws; for, in obeying their mutual attraction, they must deviate a little from the elliptical paths which they would exactly follow if they were attracted only by the sun. Not only are the planets disturbed in their motions around the sun by reason of their mutual attraction, but the satellites are also disturbed in their motions around their primaries by the attraction of the sun and by the other planets of the system. The smallness of the planetary masses in comparison with that of the sun, however, permits these bodies to conform very nearly to the laws of the elliptical motion; and hence the calculation of their perturbations is not a difficult problem. But in the motions of the satellites about their primaries, the principal disturbing body is the sun; and the effect of his attraction is too great to be overlooked. Especially is this the case in regard to the motion of the moon around the earth, the disturbing force of the sun being so great as to cause the transverse axis of her orbit to describe a complete revolution in about nine years, and also causing the nodes of the orbit to revolve in about double that interval.

2. The mathematical theory of the perturbations of the planets and satellites develops the laws according to which the various forces operate in the disturbance of each other's movements. They may be classed as follows:

First. The operation of a comparatively large force during a short interval of time;

Second. The operation of a comparatively small force during a long period of time; and

Third. A very slow change in the mean motions of the different bodies arising from the variations of the elements of their orbits produced by their mutual attractions.

The first class produces the periodic inequalities, which depend solely on the mutual distances and configurations of the different bodies; the second class produces the inequalities of long period, which may be said to depend on the configuration of the elements of their motions; and the third class produces the secular inequalities. The second and third classes affect only the mean motions.

3. The problem of deducing all the circumstances which affect the motions of the different bodies of the system, directly from the principle of universal gravitation, has, perhaps, more than any other exercised the ingenuity of mathematicians and astronomers during the last century and a half. The theories of the periodic and secular inequalities of the planets were developed in sufficient detail for the purposes of astronomy about the close of the last century. Considerable *progress* was also made in the development of the lunar theory by LA PLACE; but the magnitude of the disturbing forces being so much greater than those which disturb the planetary motions, the labor necessary for the construction of a perfect lunar theory is vastly increased. LA PLACE states that he has determined all the inequalities of the first, second and third orders, and the most important ones of the fourth order, and has continued the approximation to quantities of the fourth order inclusively, and also retained those of the fifth order, which arise in the calculation. But much more elaborate theories of the moon's motion were published during the first half of the present century, in which the approximations are carried to terms of a much higher order than was attempted by LA PLACE; the most complete and perfect, being that of PLANA, in which the approximations are carried to terms of the seventh order, was published in the year 1832. PONTÉCOULANT also investigated the lunar theory in the fourth volume of his *Théorie Analytique du Système du Monde*, which was published in the year 1846, in which he has also carried the approximations to terms of the seventh order. The theories of the moon's motion by PLANA and PONTÉCOULANT were produced by entirely different methods of development—the one by employing the true longitude as the independent variable, and the other using the mean longitude for the same purpose—and when reduced to the same form were found to be almost identically the same, the coefficients of the inequalities seldom differing from each other by so much as one second of arc. It would therefore seem that the calculations of both had been correctly made. But perhaps the most elaborate and complete investigations on the theory of the moon's motion are those of HANSEN and DELAUNAY. HANSEN published very elaborate "*Tables de la Lune*" in the year 1857; but the analysis by which he obtained his formulæ has not, so far as I am informed, yet been published. The lunar theory by DELAUNAY was published in two large quarto volumes, which appeared in the years 1860 and 1867, and is perhaps the most complete and perfect work on the lunar theory that has yet been given to astronomers.

4. In a valuable and interesting discussion on the apparent inequalities of long period in the motion of the moon, published in the *American Journal of Science and Arts* for September, 1870, Professor NEWCOMB makes the following statement in regard to the inequalities of short period in the motion of the moon:

"The problem of determining the motion of the moon around the earth under the influence of the combined attraction of the sun and planets has, more than any other, called forth the efforts of mathematicians and astronomers. Nearly every great geometer since NEWTON has added something to the simplicity or the accuracy of the solution, and in our own day we have seen it successfully com-