NOTES ON FINITE DIFFERENCES: FOR THE USE OF STUDENTS OF THE INSTITUTE OF ACTUARIES

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OF

THE INSTITUTE OF ACTUARIES.

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PREFACE.

My object in publishing these Notes on Finite Differences is to give, in a convenient form, a collection of those elementary propositions a knowledge of which is required of the students who present themselves for the Institute of Actuaries' First Examination. I have not attempted a complete account of even that small portion of the subject utilized in actuarial science, thinking it desirable to confine myself almost entirely to the methods of elementary algebra. The only instances in which a knowledge of mathematics beyond the Binomial Theorem is required of the reader are § 2, Chap. IV; Example 28; and the Note to Example 27. These are marked with asterisks, thus *.

The difference symbol has been denoted by δ when the increment of the independent variable is taken to be unity, and by Δ when this increment is not so restricted.

I am indebted to Mr. T. B. Sprague, President of the Institute of Actuaries, for some valuable suggestions, and regret that I have been able only in part to avail myself of them.

A. W. S.

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London, E.C.
20 *Feb.* 1885.

PRELIMINARY.

§ 1. As the following short account of some of the more elementary theorems of Finite Differences is intended for the use of students who have no acquaintance with the methods of analytical geometry, it seems desirable to preface it by a short explanation of the method of representing variable quantities by curved lines or diagrams.

§ 2. Let us consider the quantity x^2. Its value, of course, depends on that of x, and if any particular value be given to x, the corresponding value of x^2 may be determined by multiplication; *e.g.*, if $x=3$, $x^2=9$.

Any quantity such as x^2 which depends on x in such a manner that for each value assigned to x it takes a determinate value is called a function of x.

As another illustration of a function, consider, out of 100,000 persons born alive, the number, l_x, who would be living at age x on the assumption that these persons were subject to some definite law of mortality. Here for each value of x, the age attained, we have a definite number alive at that age. The quantity l_x is therefore, according to our definition, a function of x.

Any function of x may be conveniently denoted by the symbol u_x. With reference to u_x, the quantity x, (to which we may assign any value we please, the corresponding value of u_x being then determined), is called the independent variable. With reference to x, the quantity u_x, (whose value we consider determined when that of x is given), is called the dependent variable.

§ 3. Suppose we wish to examine the series of values which a function takes for a series of different values of the variable x which it involves. A very valuable and powerful method of conducting such an examination, and of investigating the properties of any function, is that of representing it by a diagram as follows :—

Let a straight line, which we will call OX, be drawn on a sheet of paper from a definite point O, and let lines measured along OX represent, according to some given scale, values of x. For example, if we take one inch to represent the unit of x, $x=5$ will be represented by measuring along OX a line, ON, 5 inches long.

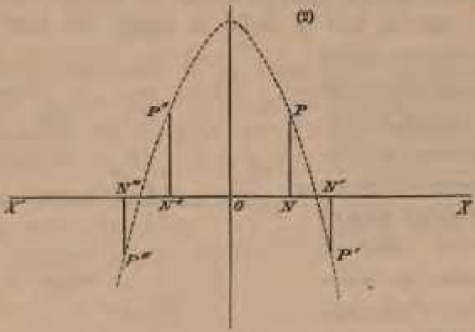
In order to represent the value of the function for any value of x, draw through the end of the line which represents the value of x a line perpendicular to OX, and of a length corresponding on the same, or, it may be, a different scale to the magnitude of the function for this value of x. For example, if the function is x^2, and the scale in each case one to the inch, for $x=2$, and therefore $x^2=4$, a line ON 2 inches long must be measured along OX and through the end of it, N, a line NP drawn perpendicular to OX and 4 inches long.

In this way, when the algebraic expression for the function is given, any number of lines such as NP may be drawn, each representing the value of the function for that value of x which is represented by the line between O and the foot of the perpendicular. The summits of these perpendiculars will form a series of points lying on a curved line, and the process above described is that of mapping out the diagram or curved line corresponding to the function.

§ 4. To complete the representation of any function of x it is necessary to provide for negative values. This is done by producing XO through O and making the convention that negative values of x shall be represented by lines measured along the produced line, which we will call OX', and negative values of the function by drawing the perpendiculars down instead of up.

It will be a useful exercise for the student to map out the diagrams for two or three different functions.

Take, for example, the function $4-x^2$. The diagram will be found to take the form shown by the dotted curved line given in Fig. (2). The curve is that known as a parabola. For the point P, x and the function are both positive;



for P' x is positive and the function negative, for P'' x is negative and the function positive, for P''' both are negative.

Lines such as ON measured along OX or OX' are usually called abscissæ, and the perpendiculars, such as PN, ordinates.

§ 5. Suppose we are given, not the algebraic expression for a function, but the curved line or diagram which forms its complete representation. Then, in order to find its value for any given value of x, we must measure along OX a line, say ON, representing x in magnitude, and draw through N a line perpendicular to ON, so as to cut the curve. If P be the point of intersection of the perpendicular and the curve, then NP will represent the value of the function corresponding to the given value of x. If this perpendicular, which may be drawn both up and down, meets the curve in more than one point, there will be more than one value of the function for the given value of x. If, on the other hand, it does not meet the curve at all, there will be no value of the function for the given value of x.

As an illustration: The curve corresponding to the function which is the square root of x may be drawn on a sheet of paper by a very simple and mechanical contrivance. Supposing it so drawn, then we can ascertain the square roots of numbers by merely measuring lines on a diagram.

The form of the curve in question is shown in Fig. (3). For each positive value of x there are two perpendiculars, PN $P'N$, equal in length but on opposite sides of OX, corresponding to