AN ELEMENTARY TREATISE ON CUBIC AND QUARTIC CURVES

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ON

CUBIC AND QUARTIC CURVES

BY

A. B. BASSET, M.A., F.R.S.
TRINITY COLLEGE CAMBRIDGE

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PREFACE.

THE present work originated in certain notes, made about twenty-five years ago, upon the properties of some of the best-known higher plane curves; but upon attempting to revise them for the press, it appeared to me impossible to discuss the subject adequately without investigating the theory of the singularities of algebraic curves. I have accordingly included Plücker's equations, which determine the number and the species of the simple singularities of any algebraic curve; and have also considered all the compound singularities which a quartic curve can possess.

This treatise is intended to be an elementary one on the subject. I have therefore avoided investigations which would require a knowledge of Modern Algebra, such as the theory of the invariants, covariants and other concomitants of a ternary quantic: and have assumed scarcely any further knowledge of analysis on the part of the reader, than is to be found in most of the ordinary text-books on the Differential Calculus and on Analytical Geometry. I have also endeavoured to give special prominence to geometrical methods, since the experience of many years has convinced me that a judicious combination of geometry and analysis is frequently capable of shortening and simplifying, what would otherwise be a tedious and lengthy investigation.

The introductory Chapter contains a few algebraic definitions and propositions which are required in subsequent portions of the work. The second one deals with the elementary theory of the singularities of algebraic curves and the theory of polar curves. The third Chapter commences with an explanation of tangential coordinates and their uses, and then proceeds to discuss a variety of miscellaneous propositions connected with reciprocal polars, the circular points at infinity and the foci of curves. Chapter IV is devoted to Plücker's equations; whilst Chapter V contains an account of the general theory of cubic curves, including the formal proof of the principal properties which are common to all curves of this degree. In this Chapter I have almost exclusively employed trilinear coordinates, since the introduction of a triangle of reference, whose elements can be chosen at pleasure, constitutes a vast improvement on the antiquated methods of homogeneous coordinates and abridged notation. Chapter VI is devoted to the consideration of a variety of special cubics, including the particular class of circular cubics which are the inverses of conics with respect to their vertices; and in this Chapter the method of Cartesian coordinates is the most appropriate. A short Chapter then follows on curves of the third class, after which the discussion of quartic curves commences.

To adequately consider such an extensive subject as quartic curves would require a separate treatise. I have therefore confined the discussion to the simple and compound singularities of curves of this degree, together with a few miscellaneous propositions; and in Chapter IX, I proceed to investigate the theory of bicircular quartics and cartesians, concluding with the general theory of circular cubics, which is better treated as a particular case of bicircular quartics than as a special case of cubic curves. Chapter X is devoted to the consideration of various well known quartic curves, most of which are bicircular or are cartesians; whilst Chapter XI deals with cycloidal curves, together with a few miscellaneous curves which frequently occur in mathematical investigations. The theory of projection, which forms the subject of the last Chapter, is explained in most treatises on Conics; but except in the case of conics, due weight has not always been given to the important fact that the projective properties of any special class of curves can be deduced from those of the simplest curve of the species. Thus all the projective properties of tricuspidal quartics can be obtained from those of the three-cusped hypocycloid or the cardioid; those of quartics with a node and a pair of cusps from the limaçon; those of quartics with three biflecnodes from the lemniscate of Bernoulli or the reciprocal polar of the four-cusped hypocycloid; whilst the properties of binodal and bicuspidal quartics can be obtained from those of bicircular quartics and cartesians.

Whenever the medical profession require a new word they usually have recourse to the Greek language, and mathematicians would do well to follow their example; since the choice of a suitable Greek word supplies a concise and pointed mode of expression which saves a great deal of circumlocution and verbosity. The old-fashioned phrase "a non-singular cubic or quartic curve" involves a contradiction of terms, since Plücker has shown that all algebraic curves except conics possess singularities; and I have therefore introduced the words *autotomic* and *anautotomic* to designate curves which respectively do and do not possess multiple points. The words *perigraphic*, *endodromic* and *exodromic*, which are defined on page 14, are also useful; in fact a word such as *aperigraphic* is indispensable in order to avoid the verbose phrase "a curve which has branches extending to infinity."

At the present day the subject of Analytical Geometry covers so vast a field that it is by no means easy to decide what to insert and what to leave out. I trust, however, that the present work will form a useful introduction to the higher branches of the subject; and will facilitate the study of a variety of curves whose properties, by reason of their beauty and elegance, deserve at least as much attention as the well-worn properties of conics.

FLEDBOROUGH HALL,
HOLYPORT, BERKS.
August, 1901.

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CHAPTER II.

THEORY OF CURVES.