AN INTRODUCTION TO MATHEMATICAL PHYSICS

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An Introduction to Mathematical Physics by R. A. Houstoun

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R. A. HOUSTOUN

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R. A. HOUSTOUN, M.A., Ph.D., D.Sc.

LECTURE ON PHYSICAL OPTICS AND ASSISTANT TO THE PROPESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF GLASGOW



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PREFACE

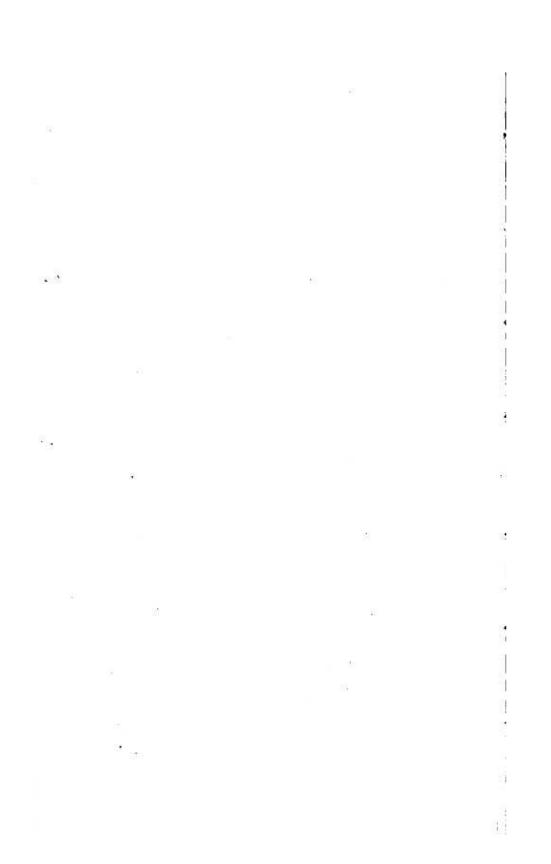
This book is the substance of lectures I have given during the past six years to the Natural Philosophy Class A in the University of Glasgow.

It is intended primarily as a class-book for mathematical students and as an introduction to the advanced treatises dealing with the subjects of the different chapters, but since the analysis is kept as simple as possible, I hope it may be useful for chemists and others who wish to learn the principles of these subjects. It is complementary to the text books in dynamics commonly used by junior honours classes.

A knowledge of the calculus and a good knowledge of elementary dynamics and physics is presupposed on the part of the student.

A large proportion of the examples has been taken from examination papers set at Glasgow by Prof. A. Gray, LL.D., F.R.S., to whom I must also express my indebtedness for many valuable suggestions. The proofs have been read with great care and thoroughness by Dr. John M'Whan of the Mathematical Department.

R. A. HOUSTOUN.



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