

# **VECTORIAL MECHANICS**

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Vectorial mechanics by L. Silberstein

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**L. SILBERSTEIN**

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# VECTORIAL MECHANICS

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## PREFACE

THE main object of this little volume is to present the chief principles and theorems of theoretical mechanics in the language of vectors, and thereby to contribute to the diffusion of the use of vectorial methods. No space has been devoted therefore to any discussion of the philosophical aspect or of the origin of such concepts as 'mass,' 'force,' 'work,' and so on, or to any description of the long and laborious route which, in the historical development of mechanics, has led to the fundamental principles of this branch of science, and especially to d'Alembert's Principle, nor of those routes which, in modern treatises, are supposed to lead to them. I emphasize d'Alembert's Principle rather than any other equivalent to it, because it is from this that a start is here made.

If our object were simply and solely a translation of mechanics into the language of vectors, we could indeed begin anywhere. But this book is not intended merely to present a loose juxtaposition of mechanical theorems and of their vectorial formulæ. On the contrary, after the enunciation, in the first section of Chapter II., of the principle mentioned already we shall be able to proceed by a continuous, deductive road, so that those readers who are acquainted with little more than d'Alembert's Principle will find here an almost systematic exposition of the chief parts of mechanics. Again, other readers who know the subject thoroughly, but only in its Cartesian form, which is based on the consideration of the scalar components of displacements, velocities, forces and so on, may perhaps wish to see their knowledge translated into vectorial language, which—to say the least—is considerably shorter and more satisfactory to the imagination than the scalar language.

In this work, only such problems will be treated as are not beyond the region proper to the vector language; and consequently

everything which has no space-directional character will, with few exceptions, be omitted.

Those readers, who are already acquainted with the elementary parts of vector algebra and analysis, may proceed at once to Chapter II.; to those who are not, may be recommended 'The elements of vectorial algebra and analysis' in Chapter III., Vol. I., of Oliver Heaviside's excellent book: *Electromagnetic Theory*, London, 1893, or E. B. Wilson's *Vector Analysis*, etc., founded upon lectures of J. W. Gibbs, New York and London, 1902, or the two sections of our Chapter I., in the first of which I have endeavoured to develop the fundamental concepts of *Vector Algebra*, and in the second the most important of the *Differential and Integral Properties of Vectors*. These sections taken together will contain, I hope, all that is needed for our mechanics, and possibly also for general Mathematical Physics.

Chapters II.-VI., of which a certain part formed the subject of a series of articles by the author, published a few years ago in a Warsaw weekly (*Przegląd Techniczny*, i.e. 'Technical Review'), contain the *General Principles* of mechanics and their consequences, e.g. the three *Special Principles* and the essential part of the dynamics of *Rigid* and of *Deformable Bodies*, closing with *Hydrodynamics*.

The not very numerous collection of *Problems and Exercises* may be useful in connexion with the various chapters, and the *Appendix* containing a kind of Vectorial-Cartesian dictionary to the whole volume may be helpful to freshmen in vectorial language.

I gladly take the opportunity of expressing my sincere thanks to my friend Prof. A. M. Worthington, C.B., F.R.S., to Mr. J. F. M'Kean of the Royal Naval College, Dartmouth, to Profs. I. J. Schwatt and George H. Hallett of the University of Pennsylvania, Philadelphia, and to Profs. Alfred W. Porter, F.R.S., and R. A. Gregory, London, for their kindness in reading the MS. and the proofs and to the Publishers for the care they have bestowed on my work.

I. S.

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