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EDINBURGH MATHEMATICAL SOCIETY.

EIGHTH SESSION, 1889-90.

First Meeting, 8th November 1889.

GEORGE A. GIBSON, Esq., M.A., President, in the Chair.

For this Session the following Office-Bearers were elected:—

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Green's and allied theorems: a historical sketch.

By GEORGE A. GIBSON, M.A.

[ABSTRACT.]

The chief purpose of the paper was to indicate the rise of transformations of the type $\iiint \frac{dV}{dx} dxdydz = \int V \cos a \, dS$ where the integral in the first member of the equation is taken throughout a closed surface, and that in the second member over the surface, a being the angle made with the axis of x by the normal to the element dS drawn outwards. It is on this transformation the analytical proof of Green's theorem depends, and it was shown to have been employed in various forms by Poisson, Duhamel, Gauss, and others, before Green's essay was generally known on the Continent. It may be observed that the essay was published at Nottingham in 1828, and seems to have been unknown to continental mathematicians till its reprint in *Crelle's Journal*, vols. 39 (1850), 44 (1852), and 47 (1854). The following references were given in the paper:—

Lagrange, in the *Mécanique Analytique* (2nd edition, 1811), Part I., sect. vii., arts. 29, 30, gives the transformation :—

$$\sum \lambda'(\delta x'.dydz + \delta y'.dzdx + \delta z'.dxdy) = \sum \lambda'(\cos a'.\delta x' + \cos \beta'.\delta y' + \cos \gamma'.\delta z')ds^2,$$

ds^2 being an element of surface.

Laplace, in the *Supplément à la Théorie de l'Action Capillaire* (which forms a supplement to Liv. x., Part II., of the *Mécanique Céleste*, published 1806)—*Oeuvres Complétes*, 1880, t4. pp. 428–432 —transforms the integral

$$\iint dxdy \left(\frac{d}{dx} \cdot \frac{p}{\sqrt{1+p^2+q^2}} + \frac{d}{dy} \frac{q}{\sqrt{1+p^2+q^2}} \right)$$

taken over the area of a section of a cylinder, whose generators are parallel to the axis of z, into the integral $\pm \int \frac{pdy - qdx}{R}$ taken along the boundary of the section, the + sign holding for the part of the curve convex to the axis of x, the − sign for that concave to the same axis.

Gauss gives a series of remarkable theorems, closely related to the transformation in question, in the introductory articles of his