A KEY TO THE NATIONAL ARITHMETIC, EXHIBITING THE OPERATION OF THE MORE DIFFICULT QUESTIONS IN THAT WORK; FOR THE USE OF TEACHERS ONLY

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649042708

A Key to the National Arithmetic, Exhibiting the Operation of the More Difficult Questions in That Work; For the Use of Teachers Only by Benjamin Greenleaf

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

BENJAMIN GREENLEAF

A KEY TO THE NATIONAL ARITHMETIC, EXHIBITING THE OPERATION OF THE MORE DIFFICULT QUESTIONS IN THAT WORK; FOR THE USE OF TEACHERS ONLY

Trieste

A

KEY

TO THE

NATIONAL ARITHMETIC,

EXHIBITING THE OPERATION OF

THE MORE DIFFICULT QUESTIONS

IN THAT WORK;

FOR THE USE OF TEACHERS ONLY.

By BENJAMIN GREENLEAF, A. M.,
AUTHOR OF THE "COMMON SCHOOL ARITHMETIC," "ALGEBRA," ETC.

NEW ELECTROTYPE EDITION.

BOSTON:
ROBERT S. DAVIS & CO.
NEW YORK: BAKER, PRATT, & CO., 149 & 141 GRAND STREET.
PHILADELPHIA: J. A. BANCROFT & COMPANY.
ST. LOUIS: GRAY, BAKER, & COMPANY.
CHICAGO: JANSEN, McCLURG, & CO.
1875.

EducT 118.75.441 LIBRARY

GREENLEAF'S SERIES OF MATHEMATICS.

1. NEW PRIMARY ARITHMETIC; on, MENTAL ARITHMETIC, on the Inductive Plan; with Easy Exercises for the Slate. Designed for Primary Schools. 84 pp.

2. NEW INTELLECTUAL ARITHMETIC, on the Inductive Plan; being an Advanced Intellectual Course, for Common Schools and Academies. 180 pp.

3. COMMON-SCHOOL ARITHMETIC; on, INTRODUCTION TO THE NATIONAL ARITHMETIC. A Complete Treatise. Improved electrotype edition. 224 pp.

4. THE NATIONAL ARITHMETIC, being a complete Course of Higher Arithmetic, for advanced Scholars in Common Schools, High Schools, and Academies. New electrotype edition, with additions and improvements. 444 pp.

5. NEW ELEMENTARY ALGEBRA; in which the First Principles of Analysis are progressively developed and simplified. For Common Schools and Academies. 224 pp.

6. NEW HIGHER ALGEBRA; an advanced Analytical Course, for High Schools, Academies, and Colleges. 294 pp. [*Just Published.*]

7. ELEMENTS OF GEOMETRY, with Practical Applications to Mensuration. 320 pp.

8. ELEMENTS OF TRIGONOMETRY, with Practical Applications and Tables.

9. ELEMENTS OF GEOMETRY AND TRIGONOMETRY; or the last two named works in one volume. 490 pp.

10. TREATISE ON SURVEYING AND NAVIGATION; with Practical Applications and Tables. [*In Preparation.*]

☞ KEYS to the ARITHMETICS, ALGEBRAS, GEOMETRY AND TRIGONOMETRY. *For Teachers only.* 6 volumes.

Entered according to Act of Congress, in the year 1837, by
BENJAMIN GREENLEAF,
in the Clerk's Office of the District Court of the District of Massachusetts.

Entered according to Act of Congress, in the year 1857, by
BENJAMIN GREENLEAF,
in the Clerk's Office of the District Court of the District of Massachusetts

Entered according to Act of Congress, in the year 1865, for renewal, by
LUCRETIA K. GREENLEAF, widow, and BETSEY P. KENDALL, EMILY A. TEWKSBURY, and LYDIA K. SYKES, children, of BENJAMIN GREENLEAF,
in the Clerk's Office of the District Court of the District of Massachusetts.

RIVERSIDE, CAMBRIDGE:
PRINTED BY H. O. HOUGHTON AND COMPANY.

PREFACE.

THE object of the author, in this publication, is to aid the teacher in communicating instruction to his pupils, and in detecting any error which they may have made in the operation of the examples.

Every instructor who has a large number of scholars under his care is aware that it is a great tax on his time, especially when in school, to examine the operation of many arithmetical questions; whereas, by the aid of a Key, he may readily detect any mistake in the operation. Besides, amid the labors of the school-room, it is often very difficult for the most able arithmetician to recollect, at the moment, all the principles involved in the solution of difficult questions; but, by recurring to a Key, this difficulty will be obviated.

The author would recommend to teachers never to point out *directly* to the pupil the method of solving a problem, nor perform the labor for him; but suggest and explain such principles as will enable him to perform the question himself.

The answers to all the examples in the Arithmetic are inserted in the Key, for the convenience of those teachers who may prefer to use the edition of the Arithmetic which does not contain the answers.

B. GREENLEAF.

BRADFORD, MASS., *September*, 1857.

CONTENTS.

	PAGE
Numeration,	5
Addition,	5
Subtraction,	6
Multiplication,	7
Division,	8
Cancelation,	9
Contractions in Division,	12
Problems,	14
United States Money,	16
Addition of United States Money,	17
Subtraction of " "	17
Multiplication of " "	17
Division " " "	17
Practical Questions by Analysis,	18
Bills,	18
Ledger Accounts,	20
Reduction of Compound Numbers,	29
Addition of Compound Numbers,	38
Subtraction of Compound Numbers,	39
Multiplication of "	40
Division " "	42
Principles and Applications,	43
Difference of Latitude,	45
Difference of Longitude,	46
Longitude and Time,	46
Examples by Analysis,	49
Factoring,	51
Divisors or Measures,	52
Common Divisors,	53
Greatest Common Divisor,	54
Least Common Multiple,	54
Common Fractions,	57
Common Denominator,	59
Addition of Common Fractions,	59
Subtraction of "	60
Multiplication of "	62
Division " "	66
Reduction of Complex Fractions,	70
Proposed Numerator or Denominator,	75
Common Numerator,	75
Greatest Common Divisor,	76
Least Common Multiple of Fractions,	77
Reduction of Denominate Fractions,	78
Addition of Denominate Fractions,	82
Subtraction of " "	83
Examples by Analysis,	87
Decimal Fractions,	90
Addition of Decimals,	91
Subtraction of Decimals,	92
Multiplication of Decimals,	93
Contractions in Multiplication of Decimals,	93
Division of Decimals,	94
Contractions in Division of Decimals,	94
Reduction of Decimals,	94
Infinite or Circulating Decimals,	103
Transformation of Repetends,	104
Addition of Circulating Decimals,	104
Subtraction of Circulating Decimals,	106
Multiplication of Circulating Decimals,	107
Continued Fractions,	108
Ratio,	111
Analysis by Ratio,	114
Simple Proportion,	118
Compound Proportion,	122
Conjoined Proportion,	125
Percentage,	130
Interest,	135
Promissory Notes,	142
Compound Interest,	146
Discount and Present Worth,	151
Banking,	153
Stocks,	156
Brokerage and Commission,	157
Account of Sales,	158
Profit and Loss,	158
Partnership,	161
Bankruptcy,	166
Taxes,	167
General Average,	168
Equation of Payments,	169
Average of Accounts,	173
Account of Storage,	177
Insurance,	180
Life Insurance,	181
Custom-House Business,	182
Coins and Currencies,	183
Exchange,	184
Arbitration of Exchange,	186
Alligation,	188
Involution,	193
Evolution,	194
Extraction of the Square Root,	194
Extraction of the Cube Root,	194
Extraction of any Root,	196
Horner's Method of Extracting Roots,	196
Application of Powers and Roots,	199
Arithmetical Progression,	209
Geometrical Progression,	210
Annuities,	212
Permutations and Combinations,	213
Analysis by Position,	214
Scales of Notation,	220
Duodecimals,	223
Mensuration,	226
Solids,	232
Tonnage,	236
Miscellaneous Examples,	237

KEY

TO

GREENLEAF'S ARITHMETIC.

NUMERATION.

1.	(Art. 40, p. 22.)	29
2.		407
3.		23,007
4.		5,000,027
5.		7,205,005
6.		2,207,604,009
7.		105,909,308,201
8.		9,000,000,008,000,000,046
9.		15,000,000,000,031,000,017
10.		507,000,000,000,203,000,057,000,018
11.		9,000,000,000,000,000,047,007,002,000,392
12.		15,000,000,000,000,000,000,000,000,010,127,026,320,426

NOTE. — The above is the French method.

ADDITION.

(ART. 47, p. 26.)

17.	80,530	24.	300,000	31.	3,837,156
18.	31,643	25.	264,088	32.	150
19.	26,798	26.	357,477	33.	26,199
20.	28,578	27.	276,605	34.	264
21.	34,383	28.	3,980,839	35.	4,801,393
22.	29,340	29.	4,183,478	36.	5,067,696
23.	283,649	30.	31,881,050	37.	5,640,426

38.	4,344,737	42.	62,075	46.	119
39.	4,935,497	43.	10,601	47.	$228
40.	1,937,678	44.	11,087	48.	$37,443
41.	118,106	45.	$82,871		

(ART. 48, p. 28.)

2.	296	4.	25,976	6.	936,318
3.	1,832	5.	643,822	7.	23,191,876

SUBTRACTION.

4.	(ART. 52, p. 31.)	5,676	24.		408,881,883,715
5.		5,119	25.		61,475,423
6.		4,409	26.		999,999
7.		2,589	27.		1
8.		48,447	28.		6,686,136
9.		46,698	29.		760,702,380
10.		17,672	30.		31,309,891
11.		53,859	31.		16,680,605
12.		411,001	32.		10,014,098,379
13.		426,944	33.		85
14.		6,202,102	34.		110
15.		799,081	35.		190
16.		1,439	36.		993,044
17.		92,690	37.		$11,810
18.		243,334	38.		
19.		617,441	39.		173
20.		900,981	40.		1,026
21.		98,999,080	41.		4,004
22.		788,889	42.		45 and 38
23.		9,393,239,896,461	43.		519,853,026

1.	(ART. 53, p. 33.)	138	5.		1,237,311
2.		525 dollars.	6.		2,500,000
3.		389 dollars.	7.		49,632 dollars.
4.		9,187 dollars.	8.		572,206 dollars.

MULTIPLICATION.

5.	(Art. **63**, p. **40.**) 3,156,492	26.	59,784
6.	6,172,835	27.	3,545,304
7.	1,979,796	28.	584,720,181,340
8.	30,316,704	29.	594,731,545
9.	16,294,896	30.	119,109,094,835
10.	22,204,188	31.	406,781,410,014
11.	45,005,091	32.	318,697,622,634
12.	77,377,566	33.	230,896,467,247
13.	293,468,329	34.	137,260,338,494
14.	161,539,842	35.	213,255,462,816
15.	274,135,320	36.	395,018,272
16.	17,247,986,832	37.	70,136,114,040
17.	$26,645	38.	475,065,601,536
18.	$5,529	39.	20,406,081,008,060,402
19.	$2,779	40.	915,527,086,788,307
20.	$21,053	41.	454,115,186,861,492
21.	13,505	42.	12,082,109,124,168,028
22.	24,386	43.	81,000,108,000,036
23.	$4,886	44.	52,370,625
24.	4,888	45.	114,972
25.	9,021	46.	29,657,416,470,704

(Art. **64**, p. **42.**)

2.	252,801	5.	2,639,559,272
3.	$11,025	6.	897,264
4.	2,784		

(Art. **64**, p. **43.**)

3.	18,190	8.	63,126,063,000
4.	410,600	9.	3,720
5.	70,000,000	10.	$888,000
6.	9,594,000,000	11.	$2,050,000
7.	700,000,000	12.	2,850,000,000