

**THE EARTH'S MOTION OF
ROTATION INCLUDING THE
THEORY OF PRECESSION
AND NUTATION**

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The Earth's Motion of Rotation Including the Theory of Precession and Nutation by C. H. H. Cheyne

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PRECESSION AND NUTATION.

BY

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PREFACE.

IN offering to the student a treatment of the Problem of the Earth's rotation somewhat different from that which has been usually given in elementary text-books, a few words of explanation are necessary.

The FIRST PART consists of an application of the method of the variation of elements to the general problem of rotation. That the formulæ for calculating these elements are identical in the motions both of translation and rotation, appeared so remarkable, that it might be well to present the latter in a form easily accessible. As far as I am aware, an elementary investigation of these formulæ has not yet been given: in attempting to supply this, I have adopted a method somewhat similar to that which I have given for the corresponding equations of the motion of translation, in an *Elementary Treatise on the Planetary Theory*. The striking analogy, thus developed, between the solutions of problems, in appearance so dissimilar, may, I hope, lead to a more complete study of Lagrange's beautiful theory of the variation of arbitrary constants.

In the SECOND PART the general rotation-formulæ are applied to the particular case of the Earth. These formulæ

afford a simple and accurate proof of the important theorem of the Stability of the Earth's axis and of the motion about it, so far as these depend upon the attractions of distant bodies. In this I have followed M. de Pontécoulant. The remaining pages are devoted to a consideration of the motion of the Earth's axis in space. In this I have obtained the formulæ for calculating Precession and Nutation, first, by an application of the general method, and afterwards, by an independent process; but I have not carried the approximation further than has been usual in elementary text-books.

C. H. H. CHEYNE.

1, DEAN'S YARD,
WESTMINSTER,
September, 1867.

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THE EARTH'S MOTION OF ROTATION.

PART I.

GENERAL THEORY OF ROTATION.

1. IN that part of Physical Astronomy which usually goes by the name of the Planetary Theory we are concerned with the motions of translation only of the planets in space: we now propose to consider their motions of rotation. The principles of the conservation of the motions of translation and rotation permit us to consider these separately, and to treat the latter as if the centre of gravity were a fixed point. We shall adopt a method perfectly rigorous, and free from all assumptions, with the single exception of the hypothesis, already required in the Planetary Theory, that the attracting bodies are so distant that their action may be supposed the same as it would be if their whole masses were condensed into their centres of gravity. Thus we shall obtain, for the determination of the motion, formulæ applicable to the case of any planet or other rigid body: an interesting application of these will then be afforded by the special circumstances which occur in the *Earth's Motion of Rotation*.

2. If the planets were exactly spherical in shape, it is clear that the attractions of the Sun, Moon, and of the other planets could produce no effect upon their rotation, since they would all pass through the centre of gravity. But