

**ELEMENTS OF DYNAMIC: AN  
INTRODUCTION TO THE  
STUDY OF MOTION AND REST  
IN SOLID AND FLUID BODIES**

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Elements of dynamic: an introduction to the study of motion and rest in solid and fluid bodies by  
W. K. Clifford

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**W. K. CLIFFORD**

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ELEMENTS OF DYNAMIC.



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# ELEMENTS OF DYNAMIC

AN INTRODUCTION TO THE STUDY OF

MOTION AND REST

IN SOLID AND FLUID BODIES

BY

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