AN INTRODUCTION TO THE USE OF GENERALIZED COÖRDINATES IN MECHANICS AND PHYSICS

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WILLIAM ELWOOD BYERLY

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PREFACE

This book was undertaken at the suggestion of my lamented colleague Professor Benjamin Osgood Peirce, and with the promise of his collaboration. His untimely death deprived me of his invaluable assistance while the second chapter of the work was still unfinished, and I have been obliged to complete my task without the aid of his remarkably wide and accurate knowledge of Mathematical Physics.

The books to which I am most indebted in preparing this treatise are Thomson and Tait's "Treatise on Natural Philosophy," Watson and Burbury's "Generalized Coördinates," Clerk Maxwell's "Electricity and Magnetism," E. J. Routh's "Dynamics of a Rigid Body," A. G. Webster's "Dynamics," and E. B. Wilson's "Advanced Calculus."

For their kindness in reading and criticizing my manuscript I am indebted to my friends Professor Arthur Gordon Webster, Professor Percy Bridgman, and Professor Harvey Newton Davis.

W. E. BYERLY

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