

ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS

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Elements of the differential and integral calculus by A. E. H. Love

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PREFACE

IN the last six years I have given annually a course of about twenty lectures on the Elements of the Differential and Integral Calculus to classes consisting chiefly of students of Chemistry and Engineering. The work of preparing and delivering such lectures, and of revising them from year to year, teaches the lecturer many things in regard to the nature of the difficulties which are encountered by students. He is led to depart frequently from the traditional order of the subject matter, and to devise numerous simplifications in the proofs of propositions. It soon appeared that the amount of mathematical knowledge which need be possessed by a student before attempting the Calculus is very much less than has been supposed. For example, the Binomial Theorem in Algebra and the Addition Equation in Trigonometry are quite unnecessary. This book is written with the view of making the subject more easily and generally accessible than it has been hitherto. The principles of the Differential and Integral Calculus ought to be counted as a part of the intellectual heritage of every educated man or woman in the twentieth century, no less than the Copernican system or the Darwinian theory. In order to make a beginning no previous knowledge of mathematics is needed beyond the most elementary notions of geometry, a little algebra,

including the law of indices, and the definitions of the trigonometric functions. In order to advance very far in the subject a student must advance in other branches of mathematics as well. This book is intended merely to help the reader to make a beginning. In order to render his progress as easy as possible, results with which he is supposed to be more or less familiar are recapitulated in the places where they are wanted, and formal proofs of some propositions are omitted from the text and placed in Appendices, along with certain rather abstract discussions.

Two things in the subject are, and apparently must continue to be, difficult. These are the actual integration of particular functions and the theory of the exponential function. For a reader in search of culture the practice of integration is not very important, but for a student who wishes to make use of the Calculus it is indispensable. The difficulty seems to be purely one of *technique*. The best that I can do to meet it is to lead up gradually to the appropriate methods, and to illustrate them by sufficiently numerous and sufficiently easy examples. The student must not allow himself to be discouraged too easily by a few failures. The theory of the exponential function, on the other hand, is essentially difficult, and the history of mathematics shows what a formidable stumbling-block it proved itself from the time of the invention of logarithms by John Napier to the time of the revision of the foundations of mathematical analysis by Augustin Louis Cauchy. For the purpose of a study of the elements of the Calculus the whole of the theory is not required, for example, the exponential theorem is unnecessary, but it is necessary either to prove or to assume at least one

proposition the rigorous proof of which is difficult. My plan has been to assume the existence of the exponential limit. This limit presents itself naturally in the process of differentiating a logarithm, while logarithms arise naturally, though not historically, from the use of indices of powers. It would be unsatisfactory to assume the existence of the limit without explanation, and for this reason an arithmetical argument has been given which makes it appear probable that the limit exists. Such arguments are often more convincing than formal proofs. It would be unsatisfactory to substitute such an argument for a proof, and merely irritating to refer the reader to a proof in some book which he may not possess, and a formal proof is given in an Appendix (pp. 191—194). I have not tried to select one which is brief, or easy to reproduce in examinations, but my choice was guided by the wish to use none but the simplest mathematical material.

The student who wishes to master the subject within the range of this book is recommended to work the Examples. In working some of these a book of Tables of mathematical functions is needed, and it is assumed that the reader knows how to compute by means of logarithms. This knowledge is not, however, required in order to read the greater part of the text.

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A. E. H. LOVE.

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