GEOMETRIC PROPERTIES COMPLETELY CHARACTERIZING THE SET OF ALL THE CURVES OF CONSTANT PRESSURE IN A FIELD OF FORCE

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Geometric Properties Completely Characterizing the Set of All the Curves of Constant Pressure in a Field of Force

BY

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Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Faculty of Pure Science, Columbia University

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CURVES OF CONSTANT PRESSURE

INTRODUCTION

In the Princeton Colloquium lectures, 1909, Professor Edward Kasner of Columbia University pointed out several unfinished problems connected with a field of force. He showed that the trajectories whose characteristics he had previously described (Transactions of the American Mathematical Society, Vol. 7, No. 3, pp. 401–424, July, 1906) might be considered as a special case of either of two more general problems: to find curves along which a constrained motion is possible such that the pressure of the moving particle against the curve is (1) proportional to the normal component of the force or (2) constant.

The pressure, since the curve is considered smooth, is connected with the normal component of acceleration by the formula $P=\frac{v^2}{r}-N$. In the case of trajectories a particle moves freely under the action of a force which depends only on the position of the particle; that is, there is no pressure and $P=o$. $P=o$ is obtained when $k=o$ from $P=k\,N$, which represents the first general problem, or when $c=o$ from $P=c$, which represents the second general problem.

Regarding $P=o$ as a special case of $P=k\,N$, Professor Kasner stated five properties characterizing the system S_k of ∞^3 curves corresponding to any value of the parameter k. Sarah Elizabeth Cronin in her dissertation, 1917, found geometric properties completely characterizing the system of ∞^4 curves obtained by combining all the systems S_k.

It is my purpose to consider the problem represented by $P=c$, the problem of curves along which a constrained motion is possible such that the pressure against the curve remains constant.

I. I shall prove that the system S_c of ∞^3 curves of constant pressure corresponding to any one value of the parameter c has four properties.

Property 1. For any given lineal element (x, y, y') the foci of the osculating parabolas of the single infinity of curves determined by the given element lie on a circle passing through the given point.

Property 2. Of the ∞^1 curves having the given lineal element $(x, y, y'_,)$ one has contact of the third order with its circle of curvature. The locus of centers of the ∞^1 hyperosculating circles obtained by varying the initial direction is a conic passing through the given point in the direction of the force acting at that point.

Property 3. The circle that corresponds according to Property 1 to a given lineal element is so situated that the element bisects the angle between the acceleration vector for the given element and the tangent to the circle at the given point.

Property 4. That curve corresponding to a given lineal element, which according to Property 2 has third order contact with its circle of curvature has a radius of curvature equal to three times the ratio of the tangential component of the force to the normal component of the space derivative of the force.

II. In the second part it will be shown that the four properties described are sufficient to determine a set of ∞^3 curves of constant pressure in a field of force.

III. By varying the parameter c we get ∞^1 sets of curves S_c and therefore ∞^4 as the total number of curves of constant pressure in a given plane field. The following properties of the quadruply infinite system are found to be sufficient to completely characterize it.

Property 1. Those curves of constant pressure which have a given curvature element have osculating conics whose centers lie on a conic tangent to the given element.

Property 2. The ∞^3 circles of the plane are curves of constant pressure and the pressure for these curves is infinite.

Property 3. The radius of curvature for the conic of Property 1 is one-tenth the difference of the radius of curvature for those curves having third order contact with their circles of curvature minus the radius of curvature of the given curvature element.

Property 4. As the curvature of a given element varies, leaving the direction and point fixed, the second center of curvature of the conic of Property 1 describes a cubic curve which passes through the given point once and has an asymptote parallel to the fixed direction with a double point on the asymptote at infinity.

Property 5. The asymptote of the cubic of Property 4 intersects the normal of the lineal element to which the cubic belongs at a point whose distance from the fixed point of the element is three-tenths the ratio of the tangential component of the force to the normal component of the space derivative of the force.

Property 6. The tangent of the angle which the cubic of Property 4 makes with the normal to the fixed element is one-tenth the sum of three ratios. The first ratio is negative three times the product of the normal component of the second space derivative of the force, as specialized for those curves which have third order contact with their circles of curvature, multiplied by the tangential component of the force divided by the square of the normal component of the first space derivative of the force. The second ratio is four times the tangent of the angle which the first space derivative of the force makes with the normal. The third is the negative tangent of the angle which the fixed element makes with the force vector.

Property 7. Of the ∞^1 curves of constant pressure having a given curvature element, two have contact of the fourth order with their osculating parabolas. The locus of the foci of these hyperosculating parabolas as the curvature varies, leaving the lineal element fixed, is a quintic curve having a triple point at the origin and having the element as double tangent. The third branch of the quintic at the origin is so placed that its tangent at that point makes with the line of force an angle bisected by the element.

Property 8. The quintic of Property 7 intersects the fixed element tangent at a point whose distance from the point of the element is five-halves the ratio of the normal component of the space derivative of the force to the normal component of the second space derivative of the force for those curves of constant pressure which are hyperosculated by the particular parabola whose focus is the point of intersection of the quintic and the element tangent.

IV. Property 1 is characteristic of any set of ∞^4 curves represented by a certain general fourth order differential equation and the other seven properties stated above are shown to be sufficient to specialize the equation and identify a set of ∞^4 curves as curves of constant pressure.

The writer takes this opportunity to gratefully acknowledge her indebtedness to Professor Kasner for his helpful criticisms and suggestions.