

**MATHEMATICAL
QUESTIONS WITH THEIR
SOLUTIONS, FROM THE
"EDUCATIONAL TIMES"**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649487691

Mathematical Questions with Their Solutions, from The "Educational Times" by W. J. C. Miller

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W. J. C. MILLER

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MATHEMATICAL QUESTIONS,

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SOLUTIONS,

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WITH MANY

Papers and Solutions not published in the "Educational Times."

EDITED BY

W. J. C. MILLER, B.A.,

REGISTERED BY

THE GENERAL MEDICAL COUNCIL.

VOL. XXX.

FROM JULY TO DECEMBER, 1878.



LONDON:

C. F. HODGSON & SON, GOUGH SQUARE,

FLEET STREET.

1878.

12753 = 2.

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$$y^2 = 3x^2 - 2(a + \beta + \gamma)x + \beta\gamma + \gamma a + a\beta,$$

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