TABLES FOR THE COMPUTATION OF THE JUPITER PERTURBATIONS OF THE GROUP OF SMALL PLANETS WHOSE MEAN DAILY MOTIONS ARE IN THE NEIGHBOURHOOD OF 750", A DISSERTATION

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TABLES FOR THE COMPUTATION OF THE JUPITER PERTURBATIONS OF THE GROUP OF SMALL PLANETS WHOSE MEAN DAILY MOTIONS ARE IN THE NEIGHBOURHOOD OF 750"

A DISSERTATION

SUBMITTED TO THE FACULTY OF THE OGDEN GRADUATE SCHOOL
OF SCIENCE IN CANDIDACY FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

(DEPARTMENT OF ASTRONOMY)

BY
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PREFACE.

The following tables, skillfully worked out by Professor D. T. WILSON of Cleveland, refer to the group of asteroids, whose mean motion is nearly $\frac{5}{2}$ of that of Jupiter, their mean motion being approximately:

$$n = 750''.$$

They are said to belong to the group $\frac{2}{5}$ of the asteroids or to the *Minerva*-type. The tables are fit to fill a space of asteroids for which the general perturbations hitherto had not been calculated.

Considering the commensurabilities of the mean motions of the small planets with regard to Jupiter, we will have the following table:

	corresponding to the commensurability
$n = 448.''7$	$2-3\,\mu$
478.6	$5-8\,\mu$
498.6	$3-5\,\mu$
523.5	$4-7\,\mu$
598.3	$1-2\,\mu$
698.0	$3-7\,\mu$
747.8	$2-5\,\mu$
797.7	$3-8\,\mu$
897.4	$1-3\,\mu$
1047.0	$2-7\,\mu$
1196.5	$1-4\,\mu$.

Of these ratios the following three are representative for the whole system of the asteroids:

		approximately
1:0	$\mu = \frac{1}{2}$	$n = 600''$
2:0	$\mu = \frac{2}{5}$	$n = 750''$
3:0	$\mu = \frac{1}{3}$	$n = 900''$,

the remaining commensurabilities either being very rare in the reality or corresponding to so high degrees of the eccentricity and of the inclination as to give remarkable types, only where the commensurabilities are very strong. Accordingly, the asteroids can in general be put in one of the three categories:

$\mu = \frac{1}{2}$	*Hecuba*-group;
$\mu = \frac{2}{5}$	*Minerva*-group;
$\mu = \frac{1}{3}$	*Hestia*-group.

The first[1] and third[2] of these groups were considered resp. by Dr. H. v. ZEIPEL and by myself and their general perturbations were carefully worked out and compared with other computations. Moreover the perturbations of Saturn[3] were calculated by Dr. H. G. BLOCK. Thus, after that the tables for the groups $\frac{2}{5}$ herewith are given by Professor D. T. WILSON[4] the computation of the general perturbations for any one of the asteroids may be worked out in a few hours in almost every case.

The tables have been used for computing the general *Jupiter*-perturbations of the following planets: (9) Metis; (32) Pomona; (29) Amphitrite[5]; (161) Athor[6]; (48) Doris[7]; (10) Hygiea[8]; (24) Themis[9]; (28) Bellona; (55) Pandora; (127) Johanna[10] and the gene-

[1] H. v. ZEIPEL. Angenäherte Jupiterstörungen für die Hecuba-Gruppe. Mémoires de l'académie impériale des sciences de S:t Petersbourg. 8 série, Vol. 12, n:o 11.

[2] K. BOHLIN. Sur le développement des perturbations planétaires. Application aux petites planètes. Astron. iakttagelser och undersökningar å Stockholms Observatorium. Band 7.

[3] H. G. BLOCK. Tafeln zur Berechnung der Störungen einer Gruppe kleiner Planeten durch Saturn. Astron. iakttagelser och undersökningar å Stockholms Observatorium. Band 8, n:o 5.

[4] D. T. WILSON. The present paper.

[5] Astron. iakttagelser och undersökningar å Stockholms Observatorium. Band 7 and Astron. Nachr., n:o 3396.

[6] Astron. Nachr., n:o 3605.

[7] Mémoires de l'académie impériale des sciences de S:t Petersbourg. 8 série, Vol. 12, n:o 11.

[8] Ibid. and in the Astron. Nachr., n:o 3793.

[9] Allgemeine Jupiter-störungen des Planeten (24) Themis. Astron. Nachr., n:o 4123 and 4551.

[10] The present memoir.

ral *Saturn*-perturbations of (32) Pomona and (29) Amphitrite.[1] In every case the agreement was satisfactory. In the several papers mentioned below all accounts for the application of the methode ougth to be given.

There are nevertheless some remarks and corrections to the matter, which should be observed and which I take this opportunity to announce.

Corrections.

1. To the »Développement des perturbations planétaires». Astron. iakttagelser och undersökningar à Stockholms Observatorium, Band 7:

page 184 note:	to be read:	$n = 0$	instead of	$n = 1$
» 83	»	$\theta_1 = \frac{w}{1-w} nz$	»	$\theta_1 = -\frac{w}{1-w} nz$
» 103	»	$-\frac{w}{1-w} g$	»	$\frac{\varkappa - w}{1-w} g$
» 105 (212)	»	$n = \frac{n_0}{1-w}$	»	$n = n_0(1-w)$
» 106	»	»	»	»
» 106	»	$-\frac{w}{1-w}$	»	$\frac{\varkappa - w}{1-w}$
» 106	»	$\varkappa = w - a_0(1-w)$	»	$\varkappa = w - \frac{1}{2} a_0(1-w)$
» 184 by $P_{11}[n-1-n-1]_{n=0}^{i=2}$		0.90309	»	0.90309_n
» 258	»	$ndz_1 = -\frac{w}{1-w} g + \cdots$	»	$ndz_1 = \frac{\varkappa - w}{1-w} g + \cdots$

2. To the »Formeln und Tafeln zur gruppenweisen Berechnung der Störungen benachbarter Planeten». Nova Acta Regiae Societatis Scientiarum Uppsaliensis, Serie 3, Vol. 17, 1896.

Page 215 to be read: for $n = 1$:	$D_{01}(n-n)_{+\pi'} = 2.6026_n$	instead of	2.699511
» » :	$D_{01}(n-n)_{-\pi'} = 1.1256$	»	1.22259_n.

Determination of the constants of the orbite.

In order to determine the elements of the orbit one may put the constants of integration for all perturbations zero and use directly the perturbations as they are given in the tables. The term in ndz:

[1] Astron. iakttagelser och undersökningar à Stockholms Observatorium. Band 8, n:o 5.

$$a_0 g = a_0 n t$$

may be omitted, as its effect is only to change the mean motion, which still is to be determined by observations or normal places. This is connected with the determination of $\varkappa$:

$$\varkappa = w - a_0(1 - w)$$

in the »Développement des perturbations planétaires» page 106. In order to employ the formulae (214), (242) and (284) pages 256—259 of the »Développement des perturbations planétaires» in the case of Professor WILSON, the constant

$$\varkappa$$

is to be changed into

$$\varkappa_1 = 2\,\varkappa,$$

because his developments, valid for $\mu_0 = \frac{2}{5}$, contain the argument $2\,\theta$ instead of θ, resp. ϑ^2 instead of ϑ, as is the case of

$$\mu_0 = \frac{1}{3} \quad \text{or} \quad \mu_0 = \frac{1}{2}.$$

Calculation of heliocentric coordinates.

Having formed the perturbations of the

Mean anomaly:	$n\delta z$
Radius vector:	ν
Third coordinate:	$s = \frac{u}{\cos i}$

the corrected mean anomaly is to be found by applying the formula:

$$nz + c_0 = nt + c_0 + n\delta z.$$

Using this corrected mean anomaly, the coordinates in the orbit:

$$r_0,\ v$$

are computed according to the elliptic formulae, and thereupon the actual radius vector is found by:

$$r = r_0(1 + \nu).$$

Rectangular heliocentric coordinates.

a) Ecliptical:

$$x = r \sin a \sin (A + u) + s \,.\, a_0 \sin i_0 \sin \Omega_0$$
$$y = r \sin b \sin (B + u) - s \,.\, a_0 \sin i_0 \cos \Omega_0$$
$$z = r \sin c \sin (C + u) + s \,.\, a_0 \cos i_0$$

where

$$\sin a \sin A = \cos \Omega_0 \qquad \sin b \sin B = \sin \Omega_0$$

$$\sin a \cos A = -\cos i_0 \sin \Omega_0 \qquad \sin b \cos B = \cos i_0 \cos \Omega_0$$

and

$$a_0$$

designates the constant value of the half great axis of the orbite;

b) Equatorial:

$$x = r \sin a \sin (A + u) + s \, . \, a_0 \cos a$$

$$y = r \sin b \sin (B + u) + s \, . \, a_0 \cos b$$

$$z = r \sin c \sin (C + u) + s \, . \, a_0 \cos c$$

where

$$\cos a = \sin i_0 \sin \Omega_0$$

$$\cos b = -[\sin i_0 \cos \varepsilon \cos \Omega_0 + \cos i_0 \sin \varepsilon] = -m \sin (\varepsilon + M)$$

$$\cos c = [-\sin i_0 \sin \varepsilon \cos \Omega_0 + \cos i_0 \cos \varepsilon] = m \cos (\varepsilon + M)$$

$$m \sin M = \sin i_0 \cos \Omega_0$$

$$m \cos M = \cos i_0$$

and a, A; b, B; c, C are the Gaussian Equatoreal Constants computed with the constant elements:

$$i_0, \; \Omega_0 .$$

Stockholm 23 September 1911.

Karl Bohlin.