AN ELEMENTARY TREATISE ON GRAPHS

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An elementary treatise on graphs by George A. Gibson

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GEORGE A. GIBSON

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PREFACE.

My object in the preparation of this text-book has been to present the subject of graphs in a connected form, simple enough in the early stages for the mere beginner while including in the ultimate development such of its more important applications as come within the range of elementary mathematics. The present tendency of mathematical teaching is perhaps to overestimate the value of graphical methods and to depreciate unduly those of analysis; but in spite of the evils attendant upon the reaction from the neglect of graphical methods, these possess, when judiciously used, a high educational value and are of essential importance to all engaged in experimental work.

From the educational point of view a graph has the great merit of representing in a simple manner the fundamental notion of functional dependence. The beginner's conceptions of a variable are usually very crude, and it is necessary that they should be clear and definite if he is to understand mathematical principles and processes; as an aid to the right comprehension of a variable, the graph renders very great service. But the graphical method may also be badly used; one of these bad uses is, in my judgment, the too common practice of plotting a graph from an insufficient number of points. The behaviour of a function, for example, in the neighbourhood of its turning values cannot be adequately understood by the beginner unless he tests it in typical cases by calculating the values of the function for a succession of values of the argument at small intervals. The process known as "cramming" is quite possible in graphical work and is less excusable there than in other departments of mathematics.

I have included, as opportunity arose, many applications of a practical kind, and I am deeply indebted to my colleagues Professors Longbottom, Maclean and Watkinson for the use of their Laboratory Note-books, on which I have drawn heavily for examples. In the text and among the Exercises examples occur which have been manufactured simply to illustrate certain processes, but examples in which the data are stated to be experimental are of course taken directly from the record of the experiments. The answers given are such as can be obtained by the methods illustrated in the text; they have been worked out by my friends Mr. John Dougall and Mr. John Miller and will be found, it is hoped, to be as accurate as the data warrant.

The Tables at the end of the book are sufficient for the calculations required in the examples; in questions on gradients however there would in some cases be an advantage in using seven-figure Tables.

Besides the gentlemen already named, my friends Dr. J. S. Mackay, Dr. A. Morgan, Mr. P. Bennett, Mr. W. A. Lindsay and Mr. P. Pinkerton have been kind enough to take an interest in the preparation of the book, and for their help in proof reading I tender them my hearty thanks. I owe a special debt of gratitude to Professor R. A. Gregory and Mr. A. T. Simmons for their advice in all matters bearing on the passage of the book through the press. The work of proof reading has however been made comparatively simple by the excellence of the printing, and I gratefully acknowledge my debt to the printing staff of Messrs. MacLehose.

GEORGE A. GIBSON.

GLASGOW, August, 1904.

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