A KEY TO THE NATIONAL ARITHMETIC, EXHIBITING THE OPERATION OF THE MORE DIFFICULT QUESTIONS IN THAT WORK

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A Key to the National Arithmetic, Exhibiting the Operation of the More Difficult Questions in that Work by Benjamin Greenleaf

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BENJAMIN GREENLEAF

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Trieste

A

KEY

TO THE

NATIONAL ARITHMETIC,

EXHIBITING THE OPERATION OF

THE MORE DIFFICULT QUESTIONS

IN THAT WORK;

FOR THE USE OF TEACHERS ONLY.

By BENJAMIN GREENLEAF, A. M.,
AUTHOR OF THE "COMMON SCHOOL ARITHMETIC," "ALGEBRA," ETC.

NEW ELECTROTYPE EDITION.

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in the Clerk's Office of the District Court of the District of Massachusetts.

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PREFACE.

THE object of the author, in this publication, is to aid the teacher in communicating instruction to his pupils, and in detecting any error which they may have made in the operation of the examples.

Every instructor who has a large number of scholars under his care is aware that it is a great tax on his time, especially when in school, to examine the operation of many arithmetical questions; whereas, by the aid of a Key, he may readily detect any mistake in the operation. Besides, amid the labors of the school-room, it is often very difficult for the most able arithmetician to recollect, at the moment, all the principles involved in the solution of difficult questions; but, by recurring to a Key, this difficulty will be obviated.

The author would recommend to teachers never to point out *directly* to the pupil the method of solving a problem, nor perform the labor for him; but suggest and explain such principles as will enable him to perform the question himself.

The answers to all the examples in the Arithmetic are inserted in the Key, for the convenience of those teachers who may prefer to use the edition of the Arithmetic which does not contain the answers.

B. GREENLEAF.

BRADFORD, MASS., *September*, 1857.

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KEY

TO

GREENLEAF'S ARITHMETIC.

NUMERATION.

1.	(Art. **40**, p. **22.**)	29
2.		407
3.		23,007
4.		5,000,027
5.		7,205,005
6.		2,207,604,009
7.		105,909,308,201
8.		9,000,000,008,000,000,046
9.		15,000,000,000,031,000,017
10.		507,000,000,000,203,000,057,000,018
11.		9,000,000,000,000,000,047,007,002,000,392
12.		15,000,000,000,000,000,000,000,010,127,026,320,426

NOTE. — The above is the French method.

ADDITION.

(ART. **47**, p. **26.**)

17.	30,530	24.	300,000	31.	3,837,156
18.	31,643	25.	264,088	32.	150
19.	26,798	26.	357,477	33.	26,199
20.	28,578	27.	276,605	34.	264
21.	34,383	28.	3,980,839	35.	4,801,393
22.	29,340	29.	4,183,478	36.	5,067,696
23.	283,649	30.	31,881,050	37.	5,640,426

38.	4,344,737	42.	62,075	46.	119
39.	4,935,497	43.	10,601	47.	$228
40.	1,937,678	44.	11,087	48.	$37,443
41.	118,106	45.	$82,871		

(ART. 48, p. 28.)

2.	296	4.	25,976	6.	936,318
3	1,832	5.	643,322	7.	23,191,876

SUBTRACTION.

4.	(ART. 52, p. 31.)	5,676	24.		408,881,883,715
5.		5,119	25.		61,475,423
6.		4,409	26.		999,999
7.		2,589	27.		1
8.		48,447	28.		6,686,136
9.		46,698	29.		760,702,380
10.		17,672	30.		31,309,891
11.		53,859	31.		16,680,605
12.		411,001	32.		10,014,098,379
13.		426,944	33.		85
14.		6,202,102	34.		110
15.		799,081	35.		190
16.		1,439	36.		993,044
17.		92,690	37.		$11,810
18.		243,334	38.		
19.		617,441	39.		173
20.		900,981	40.		1,026
21.		98,999,080	41.		4,004
22.		788,889	42.		45 and 38
23.		9,393,239,896,461	43.		519,853,026

1.	(ART. 53, p. 33.)	138	5.		1,237,311
2.		525 dollars.	6.		2,500,000
3.		389 dollars.	7.		49,632 dollars.
4.		9,187 dollars.	8.		572,206 dollars.

MULTIPLICATION.

5.	(ART. **63**, p. **40.**) 3,156,492	26.	59,784
6.	6,172,835	27.	3,545,304
7.	1,979,796	28.	584,720,181,340
8.	30,316,704	29.	594,731,545
9.	16,294,896	30.	119,109,094,835
10.	22,204,188	31.	406,781,410,014
11.	45,005,091	32.	318,697,622,634
12.	77,377,566	33.	230,896,467,247
13.	293,468,329	34.	137,260,338,494
14.	161,539,842	35.	213,255,462,816
15.	274,135,320	36.	395,018,272
16.	17,247,986,832	37.	70,136,114,040
17.	$26,645	38.	475,065,601,536
18.	$5,529	39.	20,406,081,008,060,402
19.	$2,779	40.	915,527,086,788,307
20.	$21,053	41.	454,115,186,861,492
21.	13,505	42.	12,032,109,124,168,023
22.	24,386	43.	81,000,108,000,036
23.	$4,886	44.	52,370,625
24.	4,888	45.	114,972
25.	9,021	46.	29,657,416,470,704

(ART. **64**, p. **42.**)

2.	252,801	5.	2,639,559,272
3.	$11,025	6.	897,264
4.	2,784		

(ART. **64**, p. **43.**)

3.	18,190	8.	63,126,063,000
4.	410,600	9.	3,720
5.	70,000,000	10.	$888,000
6.	9,594,000,000	11.	$2,050,000
7.	700,000,000	12.	2,850,000,000