

**ELEMENTS OF THE  
KINEMATICS OF A POINT  
AND THE RATIONAL  
MECHANICS OF A PARTICLE**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649209675

Elements of the kinematics of a point and the rational mechanics of a particle by G. O. James

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.  
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

[www.triestepublishing.com](http://www.triestepublishing.com)

**G. O. JAMES**

**ELEMENTS OF THE  
KINEMATICS OF A POINT  
AND THE RATIONAL  
MECHANICS OF A PARTICLE**



*F. Morley*

ELEMENTS  
OF THE  
KINEMATICS OF A POINT  
AND THE  
RATIONAL MECHANICS OF  
A PARTICLE

BY  
G. O. JAMES, Ph.D.  
*Instructor in Mathematics and Astronomy  
Washington University, St. Louis*

*FIRST EDITION*  
FIRST THOUSAND

NEW YORK  
JOHN WILEY & SONS  
LONDON: CHAPMAN & HALL, LIMITED  
1905

UNIV. OF CALIFORNIA  
AT LOS ANGELES

## INTRODUCTION

---

THIS book is intended for those who expect to continue the study of mechanics beyond an elementary course, and is meant to serve as an introduction to advanced treatises. For this reason special attention has been given to the principles and order of presentation, while the applications have been left almost entirely aside. No attempt has been made to avoid such mathematical terms and formulæ as seemed necessary, but those problems requiring a knowledge beyond the calculus and elementary differential equations have either been entirely omitted, or approximate solutions only have been given. Foucault's pendulum has been treated in this way. Especial attention has been given to relative motion and to motion on the Earth's surface, and to obtain a proper orientation in the subject the problems chosen have been made as general as possible.

G. O. JAMES.

WASHINGTON UNIVERSITY, May 2, 1904.

iii

463707

*H. J. Johnson - 6-26-44 - Math.*

# CONTENTS

---

## PART I

### *KINEMATICS*

#### CHAPTER I

##### THEORY OF VECTORS

ARTICLE	PAGE
1. Vectors. ....	1
2. Equipollence of two vectors and notation. ....	2
3. Geometric sum. ....	3
4. Analytic expression of geometric sum. ....	5
5. Geometric difference. ....	8
6. Analytic expression of geometric difference. ....	9
7. Decomposition of a vector. ....	9
8. Analytic expression of components. ....	10
9. Projection of a vector on a line. ....	11
10. Analytic expression of geometric sum referred to axes. ....	13
11. Geometric derivatives. ....	14
12. Projection of the geometric derivative of a vector on a plane and on an axis. ....	16
13. Projection of the geometric derivative of a vector on the vector itself. ....	19
14. Applications. ....	19

#### CHAPTER II

##### KINEMATICS OF A POINT. GENERAL THEOREMS

15. Definitions. Rest and motion. ....	22
16. Time. ....	23

ARTICLE	PAGE
17. Motion of a point. Path. . . . .	23
18. Equations of motion. . . . .	23
19. Rectilinear motion. . . . .	24
20. Units of time and of length. . . . .	25
21. Case where it is necessary to specify the units. . . . .	25
22. Homogeneous equations. . . . .	26
23. Change of units. . . . .	26
24. Uniform rectilinear motion. Velocity . . . . .	28
25. Equation of uniform motion. . . . .	29
26. Accelerated rectilinear motion. . . . .	30
27. Numerical examples. . . . .	30
28. Knots and nautical miles. . . . .	31
29. Equation of accelerated motion in terms of the mean velocity. . . . .	32
30. Acceleration. . . . .	32
31. Equation of uniformly accelerated motion. . . . .	33
32. Discussion of the motion by means of the equation of motion . . . . .	34
33. Case where the displacement is taken as the independent variable . . . . .	35

### CHAPTER III

#### APPLICATIONS TO ORDINARY RECTILINEAR MOTIONS

##### A. UNIFORM MOTION

34. Properties of uniform motion. . . . .	37
35. Problem. . . . .	38

##### B. UNIFORMLY ACCELERATED MOTION

36. Properties of uniformly accelerated motion. . . . .	39
37. Remark. . . . .	42
38. Equations obtained by taking the space described as independent variable. . . . .	42

##### C. PERIODIC MOTIONS

39. Equations of periodic motion. . . . .	43
40. Equations of harmonic motion. . . . .	44
41. Amplitude, elongation, frequency, argument, phase. . . . .	45
42. Velocity and acceleration. . . . .	47
43. Discussion of the motion. . . . .	47
44. Influence of the phase. . . . .	49



## CHAPTER IV

## RELATIVE MOTION ALONG A LINE

ARTICLE	PAGE
45. Relative motion. . . . .	51
46. Equation of relative motion. . . . .	52
47. Relative velocity and acceleration. . . . .	52
48. Apparent motion. . . . .	53
49. Convective motion. . . . .	54
50. Simultaneous motions. . . . .	55
51. Composition of harmonic motions of same period. . . . .	56
52. Representation of the amplitude and phase by a vector. . . . .	57
53. Equation of the resultant motion. . . . .	58
54. Vibrations of different periods. . . . .	59

## CHAPTER V

## VELOCITY AND ACCELERATION IN CURVILINEAR MOTION

## A. VELOCITY

55. Displacement. . . . .	61
56. Velocity. . . . .	62
57. Algebraic value of the velocity. . . . .	63
58. Projection of the velocity on the displacement. . . . .	64
59. Geometric representation of the path. . . . .	65
60. Projection of the velocity on any axis. . . . .	65

## B. ACCELERATION

61. Acceleration. Hodograph. . . . .	66
62. Projection of the acceleration on any axis. . . . .	67
63. Projection of the acceleration on the tangent and principal normal. . . . .	67
64. Projection of the acceleration on the binormal. . . . .	68
65. Condition that the acceleration be constantly tangential or normal to the path. Condition that the acceleration be constantly zero. . . . .	69
66. Composition of motions along the same path. . . . .	69

## CHAPTER VI

## ANGULAR AND AREAL MOTION. EQUATIONS AND GENERAL THEOREMS

ARTICLE	PAGE
67. Angular displacement. . . . .	70
68. Angular velocity . . . . .	71
69. Relation between linear and angular velocities. . . . .	72
70. Angular acceleration. . . . .	72
71. Relation between linear and angular accelerations. . . . .	73
72. Areal velocity. . . . .	74
73. Extension of theorems already found. . . . .	76
74. Equations of motion. Units of time and displacement. Homogeneity. . . . .	76
75. Equation of uniform motion. . . . .	77
76. Equation of uniformly accelerated motion. . . . .	78
77. Equation of motion when angular acceleration is not constant . . . . .	78
78. Properties of uniformly accelerated angular motion. . . . .	78
79. Periodic angular motion. Harmonic motion. . . . .	79
80. Relative angular motion. . . . .	80
81. Composition of harmonic angular motions of same period. . . . .	82

## CHAPTER VII

## MOTION REFERRED TO COORDINATE AXES

82. Important remark. . . . .	83
83. Projection of the motion on a plane and on an axis. . . . .	83
84. Projection of the velocity and acceleration. . . . .	84
85. Equations of motion referred to the coordinate axes. . . . .	85
86. Equations of the projected motion on the coordinate planes. . . . .	86
87. Projection of the path. . . . .	86
88. Equations of the projected motion on the coordinate axes. . . . .	87
89. Projections of the velocity. . . . .	87
90. Equations of motion of the point which describes the hodograph . . . . .	88
91. Projections of the acceleration on the coordinate axes. . . . .	88
92. Résumé. . . . .	89
93. Most general motion in which the projected motions are uniformly accelerated. . . . .	90
94. Motion in which the projected motions are harmonic. . . . .	91
95. Rectilinear harmonic motion considered as the projection of uniform circular motion. . . . .	93

## CHAPTER VIII

## RELATIVE MOTION. MOVING AXES

ARTICLE	PAGE
96. Fixed and moving axes. . . . .	95
97. Equations of absolute and relative motion. . . . .	96
98. Statement of the problem. . . . .	96
99. Motion of the relative axes. . . . .	96
100. Solution of the problem. . . . .	97

## PART II

*MECHANICS*

## CHAPTER IX

## MECHANICS OF A FREE PARTICLE

101. Material point or particle. . . . .	101
102. Purpose of this book. . . . .	102
103. Problem of mechanics. . . . .	103
104. The rôle of observation and experiment. . . . .	104
105. Principles of mechanics. . . . .	104
106. The absolute axes. Isolated particle. . . . .	104
107. First principle of mechanics. . . . .	105
108. Meaning of the first principle. . . . .	105
109. Field of force. . . . .	105
110. Uniform field of force. . . . .	106
111. Constant field of force. . . . .	106
112. Superposition of fields of force. . . . .	106
113. Second principle of mechanics. . . . .	106
114. Meaning of the second principle. . . . .	106
115. Reaction of the particle on the field. . . . .	107
116. Third principle of mechanics. . . . .	107
117. Meaning of third principle. . . . .	108
118. Properties of the coefficients $\rho_{ij}$ . . . . .	108
119. Definition of mass. . . . .	109
120. Force. . . . .	110
121. Observations on the notion of force. . . . .	110
122. Composition of forces. Resultant. . . . .	111
123. Decomposition of forces. Components. . . . .	112
124. Equilibrium of a free particle. . . . .	113