FOUNDATIONS OF FORMAL LOGIC; PP. 7-54 (NOT COMPLETE)

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649248674

Foundations of Formal Logic; pp. 7-54 (not complete) by Henry Bradford Smith

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

HENRY BRADFORD SMITH

FOUNDATIONS OF FORMAL LOGIC; PP. 7-54 (NOT COMPLETE)

Trieste

FOUNDATIONS OF FORMAL LOGIC

BY

HENRY BRADFORD SMITH

PHILADELPHIA
PRESS OF THE UNIVERSITY OF PENNSYLVANIA
1922

PREFACE

The edition of his *Primer of Logic* being exhausted, the writer is forced to prepare another outline for classroom use. Only parts of that text, however, have been retained and these have received a methodical rearrangement and expansion. Two chapters of the work, *Non-Aristotelian Logic*, appear again but modified in detail. To these have been added historical notes and citations and three new final chapters and the text has been illustrated by a number of diagrams.

The writer has again to express his indebtedness to Professor Singer for his introduction to the method which is here employed. This indebtedness is to be referred not only to the *Syllabus* of his lectures (reprinted pp. 46-52 in the writer's *Letters on Logic*) but also to many hints thrown out in private discussion.

The present work, as its title suggests, does not pretend that its system is completely developed. Its chief concern is with the foundations, upon which a theory may be built. Its solutions would have been carried farther, if the writer could have found his way through to the end.

H. B. S.

TABLE OF CONTENTS

CHAPTER I

PAGES

§ 1–6 Forms of proposition recognized by the logician.. 7–13

CHAPTER II

§ 7–11 Relations of "better" and "worse." Immediate inference.................................... 14–18

CHAPTER III

§ 12–16 Moods of the syllogism........................ 19–25

CHAPTER IV

§ 17–22 General solution of the sorites................. 26–32

CHAPTER V

§ 23–31 Foundations of the calculus..................... 33–40

CHAPTER VI

§ 32–35 General solution of the syllogism............... 41–44

CHAPTER VII

§ 36–40 On certain supposed fallacies................... 45–49

CHAPTER VIII

§ 41–42 Alternative systems. Non-Aristotelian logic.... 50–51

CHAPTER IX

§ 43–46 The classical system and its verification 52–56

CHAPTER I

§ 1. The problem of a deductive science is to define its elements, the objects of which it treats, by an enumeration of their formal properties. These properties are to be found within the system of which these objects are the parts. The task of logic[1] is, then, to develop its own system by constructing all the true and all the untrue propositions into which its elements enter exclusively.

According to Schröder, if $a, b, c \ldots$ be a set of indefinables and R a relation, then $a, b, c \ldots R$ are said to form a *system*, provided $p\,R\,q$ stand for an assertion that must be either true or false and can not be both true and false,—p and q representing any members of the set, $a, b, c \ldots$ (See *Schröder's Abriss der Algebra der Logik*, by Dr. Eugen Müller, Leipzig, 1909, p. 18.)

§ 2. We shall begin with an enumeration of the *forms* with which logic deals.

The *Categorical* forms are composed of *terms* and *relationships* and are represented by the following propositional *functions:*

$$\alpha(ab) = \textit{All a is all b},$$
$$\beta(ab) = \textit{Some a is some b},$$
$$\gamma(ab) = \textit{All a is some b},$$
$$\epsilon(ab) = \textit{No a is b}.$$

These are called propositional functions because each one is true only for some, not for all meanings of a and b; because their truth or untruth depends upon the meaning of a and b. Whenever it is desired to designate indifferently any one of these (i. e., to leave it unsettled which one is meant) the notation $x(ab)$, $y(ab)$, etc., will be employed. In the proposition $x(ab)$ the terms are the *subject* a and the *predicate* b and the *term order* is the order subject-predicate. Whenever the term order is unsettled a comma will appear between the terms. Thus $x(a, b)$ may mean either $x(ab)$ or $x(ba)$. The relationships in the categorical forms are the copula *is* and the adjectives of quantity, *all*, *some* and *no*.

[1] Logic is from λογική, an adjective with a substantive understood, and λογική is from λόγος, which denoted both the thought and its expression (*ratio* and *oratio*). This ambiguity passed into the meaning of the derivative λογική and led indirectly to a dispute as to whether Logic deals with the laws of thought or with the laws of the expression of thought.

The word *some* is ambiguous in ordinary speech, meaning either *some at least, possibly all* or *some at most, some at least, not all.* It is this latter interpretation, which we shall assume to be forced upon us by the propositions which we say shall be true or untrue in our science.

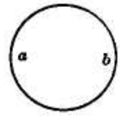
In 1846 Sir William Hamilton published the prospectus of an essay on a *New Analytic of Logical Forms*, which revived the question as to whether or not the quantity of the predicate should be made explicit. The chief difficulties of his system result from the ambiguity of the meaning of *some*, from the effort to make every form of categorical expression simply convertible and from the seemingly curious effort to establish an order of *better* and *worse* among the relations connecting subject and predicate. Four of Hamilton's eight forms are redundant. Those that are essential are represented here by the letters, α, β, γ and ϵ.

§ 3. The terms, a and b, stand for *classes*, for a group of objects conceived by the aid of a common property. Every substantive in the language is the symbol for such a group. There is a certain analogy between the manner in which closed areas overlap and the manner in which classes overlap, which was first pointed out by the mathematician Euler. This analogy often breaks down, as might be expected, for some limiting case and has perhaps led the logician astray as often as it has aided him. It will prove to be invaluable, however, in enabling us to attach a *preliminary* meaning to our symbols of relationship.

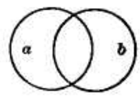
"As a general notion contains an infinite number of individual objects, we may consider it a space in which they are all contained. Thus for the notion of *man* we form a space, in which we conceive all men to be comprehended. For the notion of *mortal* we form another in which we conceive everything mortal to be comprehended. And when I affirm *all men are mortal* it is the same thing with affirming that the first figure is contained in the second. . . . These circles, or rather these spaces, for it is of no importance of what figure they are, are extremely commodious for facilitating our reflections on this subject, and for unfolding all the boasted mysteries of logic, which that art finds it so difficult to explain; whereas by means of these signs the whole is rendered sensible to the eye." (*Letters of Euler addressed to a German Princess*, by David Brewster, New York, 1846, Vol. I, pp. 339–341.)

The diagrammatic representation of the categorical forms is given below.

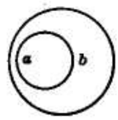
All a is all b



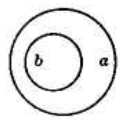
Some a is some b



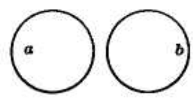
All a is some b



All b is some a



No a is b



Assuming now that these forms exhaust all of the modes in which two closed areas may overlap, it will seem natural to declare, on the ground of our analogy, that any two classes, a and b, must be related in one, and can not be related in more than one, of these five ways. The assertion that a and b must realize one of these possibilities we shall term the *propositional universe*. The assertion that a and b must realize more than one of these possibilities we shall term the *propositional null*. The former assertion will then appear to be true for all meanings of the terms and the latter assertion will appear to be untrue for all meanings of the terms.

§ 4. The remaining forms of proposition, which are recognized by the logician, are:

The *Hypothetical* form,

if x(is true) then y(is true),
$= x$ implies $y = x \angle y$,
x does not imply $y = (x \angle y)'$;

The *Conjunctive* form,

x(is true) and y(is true),
$= xy$;

The *Disjunctive* form,

either x(is true) or y(is true),
$= x + y$.

Here x and y in turn stand for any sort of proposition. If they happen to be categorical forms, then we should replace the abbreviations above by the more definite notation, $x(a, b) \angle y(a, b)$, $x(a, b)y(a, b)$, $x(a, b) + y(a, b)$. The untruth of x will be denoted by x', the untruth of x' by x''.

The relations of ***inclusion*** and ***implication*** are usually rendered by Peano's sign of a flat "C" opening to the left, the initial letter of the word *contains*; or by Schröder's half bracket opening to the right and drawn through an equality sign. The symbol of the text is a simplification of the one employed by Schröder, only the upper half being retained.

§ 5. The word "true" never connotes possibly true or true in some instances. It means *necessarily true*, true for all cases, true for all meanings of the terms. The following propositions may be verified at once "empirically" by the aid of Euler's diagrams. If the student will examine each illustration with care, he will begin to attach a meaning to the symbolism. Let him observe that the figure in each case represents the part to the left of the implication sign as true and the part to the right of the implication sign as true at the same time.

$a(ab)\ \epsilon(cb) \angle \epsilon(ca)$