STEREOTYPE EDITION. ELEMENTS OF GEOMETRY

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649571673

Stereotype Edition. Elements of Geometry by A. M. Legendre & Lohn Farrar

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

A. M. LEGENDRE & LOHN FARRAR

STEREOTYPE EDITION. ELEMENTS OF GEOMETRY

Trieste

Stereotype Edition.

ELEMENTS OF GEOMETRY,

BY

A. M. LEGENDRE,

MEMBER OF THE INSTITUTE AND THE LEGION OF HONOUR, OF THE ROYAL SOCIETY OF LONDON, &c.

TRANSLATED FROM THE FRENCH,

FOR

THE USE OF THE STUDENTS OF THE UNIVERSITY

AT

CAMBRIDGE, NEW ENGLAND,

BY JOHN FARRAR,

PROFESSOR OF MATHEMATICS AND NATURAL PHILOSOPHY

New Edition, Improved and Enlarged

BOSTON:
HILLIARD, GRAY, AND COMPANY.
1841.

Stereotyped at the
Boston Type and Stereotype Foundry.

ADVERTISEMENT.

THE work of M. LEGENDRE, of which the following is a translation, is thought to unite the advantages of modern discoveries and improvements with the strictness of the ancient method. It has now been in use for a considerable number of years, and its character is sufficiently established. It is generally considered as the most complete and extensive treatise on the elements of geometry which has yet appeared. It has been adopted as the basis of the article on geometry in the fourth edition of the Encyclopædia Britannica, lately published, and in the Edinburgh Encyclopædia, edited by Dr. Brewster.

In the original, the several parts are called books, and the propositions of each book are numbered after the manner of Euclid. It was thought more convenient for purposes of reference to number definitions, propositions, corollaries, &c., in one continued series. Moreover, the work is divided into two parts, one treating of plane figures, and the other of solids; and the subdivisions of each part are denominated sections.

As a knowledge of algebraical signs and the theory of proportions is necessary to the understanding of this treatise, a brief explanation of these, taken chiefly from Lacroix's geometry, and forming properly a supplement to this arithmetic, is prefixed to the work, under the title of an Introduction.

The parts omitted in the first edition of this translation on spherical isoperimetrical polygons, and on the regular poly-

edrons, are inserted in this, at the end of the fourth section of the second part.

Also an improved demonstration of the theorem for the solidity of the triangular pyramid, by M. Queret of St. Malo, is subjoined at the end.

But the principal improvement in this edition consists in a new demonstration of the theorem relative to the three angles of a triangle, concerning which the author remarks, that "it is perhaps the most simple and the most direct that is to be found of a purely elementary nature. We hope it will be favourably received by the lovers of geometrical exactness, and that it will redeem elementary geometry from a reproach to which the theory of parallel lines has hitherto been liable."

There is, moreover, appended to this edition a copious collection of questions, selected principally from Bland's *Geometrical Problems*, and intended as an exercise for the learner.

CAMBRIDGE, *October*, 1830.

PREFACE.

THE method of the ancients is very generally regarded as the most satisfactory and the most proper for representing geometrical truths. It not only accustoms the student to great strictness in reasoning, which is a precious advantage, but it offers, at the same time, a discipline of peculiar kind, distinct from that of analysis, and which, in important mathematical researches, may afford great assistance towards discovering the most simple and elegant solutions.

I have thought it proper, therefore, to adopt in this work the same method which we find in the writings of Euclid and Archimedes; but, in following nearly these illustrious models, I have endeavoured to improve certain points of the elements, which they left imperfect, and especially the theory of solids, which has hitherto been the most neglected.

The definition of a straight line being the most important of the elements, I have wished to be able to give to it all the exactness and precision of which it is susceptible. Perhaps I might have attained this object by calling a straight line that which can have only one position between two given points. For, from this essential property we can deduce all the other properties of a straight line, and particularly that of its being the shortest between two given points. But, in order to this, it would have been necessary to enter into subtile discussions, and to distinguish, in the course of several propositions, the straight line drawn between two points from the shortest line which measures the distance of these same points. I have preferred, in order not to render the introduction to geometry too difficult, to sacrifice something of the exactness at which I aimed. Accordingly, I shall call a *straight line* that which is the shortest between two points, and I shall suppose that there can be only one between the same points. It is upon this principle, considered at the same time as a definition and an axiom, that I have endeavoured to establish the entire edifice of the elements.

It is necessary to the understanding of this work, that the reader should have a knowledge of the theory of proportions which is explained in common treatises, either of arithmetic or algebra; he is supposed also to be acquainted with the first rules of algebra; such as the addition and subtraction of quantities, and the most simple operations belonging to equations of the first degree. The ancients, who had not a knowledge of algebra, supplied the want of it by reasoning and by the use of proportions, which they managed with great dexterity. As for us, who have this instrument in addition to what they possessed, we should do wrong not to make use of it, if any new facilities are to be deriv ed from it. I have, accordingly, not hesitated to employ the signs and operations of algebra, when I have thought it necessary; but I have guarded against involving in difficult operations what ought by its nature to be simple; and all the use I have made of algebra, in these elements, consists, as I have already said, in a few very simple rules, which may be understood almost without suspecting that they belong to algebra.

Besides, it has appeared to me, that, if the study of geometry ought to be preceded by certain lessons in algebra, it would be not less advantageous to carry on the study of these two sciences together, and to intermix them as much as possible. According as we advance in geometry, we find it necessary to combine together a greater number of relations; and algebra may be of great service in conducting us to our conclusions by the readiest and most easy method.

This work is divided into eight sections, four of which treat of plane geometry, and four of solid geometry.

The first section, entitled *first principles*, &c. contains the properties of straight lines which meet those of perpendiculars, the theorem upon the sum of the angles of a triangle, the theory of parallel lines, &c.

The second section, entitled *the circle*, treats of the most simple properties of the circle, and those of chords, of tangents, and of the measure of angles by the arcs of a *circle*.

These two sections are followed by the resolution of certain problems relating to the construction of figures.

The third section, entitled the *proportions of figures*, contains the measure of surfaces, their comparison, the properties of a right-angled triangle, those of equiangular triangles, of similar

figures, &c. We shall be found fault with, perhaps, for having blended the properties of lines with those of surfaces; but in this we have followed pretty nearly the example of Euclid, and this order cannot fail of being good, if the propositions are well connected together. This section also is followed by a series of problems relating to the objects of which it treats.

The fourth section treats of *regular polygons and of the measure of the circle.* Two lemmas are employed as the basis of this measure, which is otherwise demonstrated after the manner of Archimedes. We have then given two methods of approximation for squaring the circle, one of which is that of James Gregory. This section is followed by an appendix, in which we have demonstrated that the circle is greater than any rectilineal figure of the same perimeter.

The first section of the second part contains the properties of *planes* and of *solid angles.* This part is very necessary for the understanding of solids, and of figures in which different planes are considered. We have endeavoured to render it more clear and more rigorous than it is in common works.

The second section of the second part treats of *polyedrons* and of their measure. This section will be found to be very different from that relating to the same subject in other treatises: we have thought we ought to present it in a manner entirely new.

The third section of this part is an abridged treatise *on the sphere and spherical triangles.* This treatise does not ordinarily make a part of the elements of geometry; still we have thought it proper to consider so much of it as may form an introduction to spherical trigonometry.

The fourth section of the second part treats of *the three round bodies,* which are the sphere, the cone, and the cylinder. The measure of the surfaces and solidities of these bodies is determined by a method analogous to that of Archimedes, and founded, as to surfaces, upon the same principles, which we have endeavoured to demonstrate under the name of *preliminary lemmas.*

At the end of this section is added an appendix to the third section of the second part on *spherical isoperimetrical polygons;* and an appendix to the second and third sections of this part on the *regular polyedrons.*