THE OPEN COURT SERIES OF CLASSICS OF SCIENCE AND PHILOSOPHY, NO. 1. CONTRIBUTIONS TO THE FOUNDING OF THE THEORY OF TRANSFINITE NUMBERS

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The Open Court Series of Classics of Science and Philosophy, No. 1. Contributions to the Founding of the Theory of Transfinite Numbers by Georg Cantor & Philip E. B. Jourdain

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GEORG CANTOR & PHILIP E. B. JOURDAIN

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BY

GEORG CANTOR

TRANSLATED, AND PROVIDED WITH AN INTRODUCTION AND NOTES, BY

PHILIP E. B. JOURDAIN
M.A. (CANTAB.)

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PREFACE

THIS volume contains a translation of the two very important memoirs of Georg Cantor on transfinite numbers which appeared in the *Mathematische Annalen* for 1895 and 1897* under the title: "Beiträge zur Begründung der transfiniten Mengenlehre." It seems to me that, since these memoirs are chiefly occupied with the investigation of the various transfinite cardinal and ordinal numbers and not with investigations belonging to what is usually described as "the theory of aggregates" or "the theory of sets" (*Mengenlehre*, *théorie des ensembles*), —the elements of the sets being real or complex numbers which are imaged as geometrical "points" in space of one or more dimensions,—the title given to them in this translation is more suitable.

These memoirs are the final and logically purified statement of many of the most important results of the long series of memoirs begun by Cantor in 1870. It is, I think, necessary, if we are to appreciate the full import of Cantor's work on transfinite numbers, to have thought through and to bear in mind Cantor's earlier researches on the theory of point-aggregates. It was in these researches that the need for the

* Vol. xlvi, 1895, pp. 481–512; vol. xlix, 1897, pp. 207–246.

transfinite numbers first showed itself, and it is only by the study of these researches that the majority of us can annihilate the feeling of arbitrariness and even insecurity about the introduction of these numbers. Furthermore, it is also necessary to trace backwards, especially through Weierstrass, the course of those researches which led to Cantor's work. I have, then, prefixed an Introduction tracing the growth of parts of the theory of functions during the nineteenth century, and dealing, in some detail, with the fundamental work of Weierstrass and others, and with the work of Cantor from 1870 to 1895. Some notes at the end contain a short account of the developments of the theory of transfinite numbers since 1897. In these notes and in the Introduction I have been greatly helped by the information that Professor Cantor gave me in the course of a long correspondence on the theory of aggregates which we carried on many years ago.

The philosophical revolution brought about by Cantor's work was even greater, perhaps, than the mathematical one. With few exceptions, mathematicians joyfully accepted, built upon, scrutinized, and perfected the foundations of Cantor's undying theory; but very many philosophers combated it. This seems to have been because very few understood it. I hope that this book may help to make the subject better known to both philosophers and mathematicians.

The three men whose influence on modern pure mathematics—and indirectly modern logic and the

philosophy which abuts on it—is most marked are Karl Weierstrass, Richard Dedekind, and Georg Cantor. A great part of Dedekind's work has developed along a direction parallel to the work of Cantor, and it is instructive to compare with Cantor's work Dedekind's *Stetigkeit und irrationale Zahlen* and *Was sind und was sollen die Zahlen?*, of which excellent English translations have been issued by the publishers of the present book.*

There is a French translation† of these memoirs of Cantor's, but there is no English translation of them. For kind permission to make the translation, I am indebted to Messrs B. G. Teubner of Leipzig and Berlin, the publishers of the *Mathematische Annalen*.

PHILIP E. B. JOURDAIN.

* *Essays on the Theory of Numbers* (I, *Continuity and Irrational Numbers*; II, *The Nature and Meaning of Numbers*), translated by W. W. Beman, Chicago, 1901. I shall refer to this as *Essays on Number*.

† By F. Marotte, *Sur les fondements de la théorie des ensembles transfinis*, Paris, 1899.

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