LATITUDE DEVELOPMENTS CONNECTED WITH GEODESY AND CARTOGRAPHY WITH TABLES INCLUDING A TABLE FOR LAMBERT EQUAL-AREA MERIDIONAL PROJECTION

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OSCAR S. ADAMS

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LATITUDE DEVELOPMENTS CONNECTED WITH GEODESY AND CARTOGRAPHY

WITH TABLES

INCLUDING A TABLE FOR

LAMBERT EQUAL-AREA MERIDIONAL PROJECTION

BY

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Geodetic Computer

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FOREWORD.

There are five different kinds of latitude that come under consideration in the application of mathematical analysis to questions of geodesy and cartography. It is the aim of this publication to express the difference between the geodetic or astronomic latitude and each of the various four other kinds of latitude in a series of the sines of the multiple arcs. This difference in each case is obtained in an expression in the sines of the multiple arcs of the geodetic or astronomic latitude and also in a series of the sines of the multiple arcs of the other latitude in question.

The analysis connected with the development of both the isometric or conformal latitude [a] and of the authalic or equal-area latitude[a] is given in some degree of detail, since it is a good example of the application of mathematical analysis to such questions.

The series are derived in their general form in the first instance in which no geodetic constant appears except the eccentricity. At the end of the text in this publication the numerical values of the various coefficients are given computed for the Clarke spheroid of 1866. This is the spheroid that is used for all geodetic purposes in North America. Finally, tables are given of the results of the computations for this spheroid calculated for every half degree of latitude. These results and tables will be useful in connection with all geodetic and cartographic questions in which it is desired to take into consideration the spheroidal shape of the earth. It is believed that no previous table has been computed for the Clarke spheroid of 1866, at least none for half degrees of latitude. It is thought that the idea of the authalic latitude is new in the science of cartography. It has been applied in the computation of the elements of an Albers' equal-area projection for the United States, and it has been found materially to simplify the calculations to be performed.

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[a] For the full definition of these terms see pp. 8 and 10.

It is thought that in a later publication on equivalent or equal-area projections this latitude may be applied in the theory of the various types of projection belonging to this class.

In addition to the latitude tables there are given tables for transformation from latitude and longitude to arc distance and azimuth from a point on the equator. After these is given a table of the radial distance for a Lambert azimuthal equal-area projection upon a meridional plane, and finally a table of the coordinates for such a projection.

It is hoped that the analysis employed in the derivation of the formulas may be of interest to those who have to deal with the applications of mathematical theory to such problems as arise in practice. A few examples of such applications are of more value than any amount of the theory without the practical working out of the results in specific cases. "Learn to do by doing" is a safe maxim at all times. Finally, the numerical form of the results should appeal to those who wish to use these latitudes in questions of geodesy or cartography. The numerical forms of the expansions are given both in numbers and in logarithms. If a multiplying machine is available the coefficients expressed in numbers are more useful, but in case such a machine is not at hand it is necessary to resort to logarithms. With a five-place table of sines the results ought to be good to tenths of a second, and a six-place table should give results good to hundredths of a second. The results of computation given in the tables were derived from an eight-place table of natural sines.

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By Oscar S. Adams,
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DERIVATION OF DEFINITIONS.

In considering subjects connected with geodesy and cartography there are five different kinds of latitude that are found to be of interest and of use in practical applications. We shall now proceed to apply analysis in the derivation of the definitions of these latitudes.

If the meridian ellipse is defined by equations in the parametric form

$$x = a \cos \theta$$
$$y = b \sin \theta,$$

then θ is called the parametric latitude, a is the semimajor axis, and b is the semiminor axis of the meridian ellipse.

The geodetic or astronomic latitude is the angle which the normal at a given point of the ellipse makes with the axis of x. This latitude, denoted by φ, will then be defined analytically by the expression

$$\tan \varphi = -\frac{dx}{dy},$$

since it is perpendicular to the tangent at the point x, y. But from the parametric equations we get

$$-\frac{dx}{dy} = \frac{a \sin \theta}{b \cos \theta} = \frac{a}{b} \tan \theta.$$

Hence

$$\tan \varphi = \frac{a}{b} \tan \theta,$$

or

$$\tan \theta = \frac{b}{a} \tan \varphi.$$