EUCLID'S ELEMENTS OF GEOMETRY, THE FIRST BOOK, WITH EXPLANATORY NOTES

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ROBERTS POTTS

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ELEMENTS OF GEOMETRY,

THE FIRST BOOK,

CHIEFLY FROM THE TEXT OF DR. SIMSON,

WITH EXPLANATORY NOTES;

A SERIES OF QUESTIONS ON EACH BOOK;

AND A SELECTION OF GEOMETRICAL EXERCISES FROM THE

SENATE-HOUSE AND COLLEGE EXAMINATION

PAPERS; WITH HINTS, ETC.

BY

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CORRECTED AND IMPROVED.

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PREFACE TO THE THIRD EDITION.

SOME time after the publication of an Octavo Edition of Euclid's Elements with Geometrical Exercises, &c., designed for the use of Academical Students; at the request of some schoolmasters of eminence, a duodecimo Edition of the Six Books was put forth on the same plan for the use of Schools. Soon after its appearance, Professor Christie, the Secretary of the Royal Society, in the Preface to his Treatise on Descriptive Geometry for the use of the Royal Military Academy, was pleased to notice these works in the following terms :—
"When the greater Portion of this Part of the Course was printed, and had for some time been in use in the Academy, a new Edition of Euclid's Elements, by Mr. Robert Potts, M.A., of Trinity College, Cambridge, which is likely to supersede most others, to the extent, at least, of the Six Books, was published. From the manner of arranging the Demonstrations, this edition has the advantages of the symbolical form, and it is at the same time free from the manifold objections to which that form is open. The duodecimo edition of this Work, comprising only the first Six Books of Euclid, with Deductions from them, having been introduced at this Institution as a text-book, now renders any other Treatise on Plane Geometry unnecessary in our course of Mathematics."

For the very favourable reception which both Editions have met with, the Editor's grateful acknowledgements are due. It has been his desire in putting forth a revised Edition of the School Euclid, to render the work in some degree more worthy of the favour which the former editions have received. In the present Edition several errors and oversights have been corrected and some additions made to the notes: the questions on each book have been considerably augmented and a better arrangement of the Geometrical Exercises has been attempted: and lastly, some hints and remarks on them have been given to assist the learner. The additions made to the present Edition amount to more than fifty pages, and, it is hoped, that they will render the work more useful to the learner.

And here an occasion may be taken to quote the opinions of some able men respecting the use and importance of the Mathematical Sciences.

On the subject of Education in its most extensive sense, an ancient writer "directs the aspirant after excellence to commence with the Science of Moral Culture; to proceed next to Logic; next to Mathematics; next to Physics; and lastly, to Theology." Another writer on Education would place Mathematics before Logic, which (he remarks) "seems the preferable course : for by practising itself in the

former, the mind becomes stored with distinctions; the faculties of
constancy and firmness are established; and its rule is always to dis-
tinguish between cavilling and investigation—between *close reasoning*
and *cross reasoning*; for the contrary of all which habits, those are for
the most part noted, who apply themselves to Logic without studying
in some department of Mathematics; taking noise and wrangling for
proficiency, and thinking refutation accomplished by the instancing
of a doubt. This will explain the inscription placed by Plato over the
door of his house: 'Whoso knows not Geometry, let him not enter
here.' On the precedence of Moral Culture, however, to all the other
Sciences, the acknowledgement is general, and the agreement entire."
The same writer recommends the study of the Mathematics, for the
cure of "compound ignorance." "Of this," he proceeds to say, "the
essence is opinion not agreeable to fact; and it necessarily involves
another opinion, namely, that we are already possessed of knowledge.
So that besides not knowing, we know not that we know not; and
hence its designation of compound ignorance. In like manner, as of
many chronic complaints and established maladies, no cure can be
effected by physicians of the body : of this, no cure can be effected by
physicians of the mind : for with a pre-supposal of knowledge in our
own regard, the pursuit and acquirement of further knowledge is not
to be looked for. The approximate cure, and one from which in the
main much benefit may be anticipated, is to engage the patient in the
study of measures (Geometry, computation, &c.); for in such pursuits
the true and the false are separated by the clearest interval, and no
room is left for the intrusions of fancy. From these the mind may
discover the delight of certainty; and when, on returning to his own
opinions, it finds in them no such sort of repose and gratification, it
may discover their erroneous character, its ignorance may become
simple, and a capacity for the acquirement of truth and virtue be
obtained."

Lord Bacon, the founder of Inductive Philosophy, was not insen-
sible of the high importance of the Mathematical Sciences, as appears
in the following passage from his work on "The Advancement of
Learning."

"The Mathematics are either pure or mixed. To the pure Mathe-
matics are those sciences belonging which handle quantity determinate,
merely severed from any axioms of natural philosophy; and these are
two, Geometry, and Arithmetic; the one handling quantity continued,
and the other dissevered. Mixed hath for subject some axioms or
parts of natural philosophy, and considereth quantity determined, as it
is auxiliary and incident unto them. For many parts of nature can

neither be invented with sufficient subtlety, nor demonstrated with sufficient perspicuity, nor accommodated unto use with sufficient dexterity, without the aid and intervening of the Mathematics: of which sort are perspective, music, astronomy, cosmography, architecture, enginery, and divers others.

"In the Mathematics I can report no deficience, except it be that men do not sufficiently understand the excellent use of the pure Mathematics, in that they do remedy and cure many defects in the wit and faculties intellectual. For, if the wit be dull, they sharpen it; if too wandering, they fix it; if too inherent in the sense, they abstract it. So that as tennis is a game of no use in itself, but of great use in respect that it maketh a quick eye, and a body ready to put itself into all postures; so in the Mathematics, that use which is collateral and intervenient, is no less worthy than that which is principal and intended. And as for the mixed Mathematics, I may only make this prediction, that there cannot fail to be more kinds of them, as nature grows further disclosed."

How truly has this prediction been fulfilled in the subsequent advancement of the Mixed Sciences, and in the applications of the pure Mathematics to Natural Philosophy!

Dr. Whewell, in his "Thoughts on the Study of Mathematics," has maintained, that mathematical studies judiciously pursued, form one of the most effective means of developing and cultivating the reason: and that "the object of a *liberal education* is to develope the whole mental system of man;—to make his speculative inferences coincide with his practical convictions;—to enable him to render a reason for the belief that is in him, and not to leave him in the condition of Solomon's sluggard, who is wiser in his own conceit than seven men that *can* render a reason." And in his more recent work entitled, "Of a Liberal Education, &c." he has more fully shewn the importance of Geometry as one of the most effectual instruments of intellectual education. In page 55 he thus proceeds:—"But besides the value of Mathematical Studies in Education, as a perfect example and complete exercise of demonstrative reasoning; Mathematical Truths have this additional recommendation, that they have always been referred to, by each successive generation of thoughtful and cultivated men, as examples of truth and of demonstration; and have thus become standard points of reference, among cultivated men, whenever they speak of truth, knowledge, or proof. Thus Mathematics has not only a disciplinal but an historical interest. This is peculiarly the case with those portions of Mathematics which we have mentioned. We find geometrical proof adduced in illustration of the

nature of reasoning, in the earliest speculations on this subject, the Dialogues of Plato; we find geometrical proof one of the main subjects of discussion in some of the most recent of such speculations, as those of Dugald Stewart and his contemporaries. The recollection of the truths of Elementary Geometry has, in all ages, given a meaning and a reality to the best attempts to explain man's power of arriving at truth. Other branches of Mathematics have, in like manner, become recognized examples, among educated men, of man's powers of attaining truth."

Dr. Pemberton, in the preface to his view of Sir Isaac Newton's Discoveries, makes mention of the circumstance, "that Newton used to speak with regret of his mistake, at the beginning of his Mathematical Studies, in having applied himself to the works of Descartes and other Algebraical writers, before he had considered the Elements of Euclid with the attention they deserve."

To these we may subjoin the opinion of Mr. John Stuart Mill, which he has recorded in his invaluable System of Logic, (Vol. II. p. 180) in the following terms. "The value of Mathematical instruction as a preparation for those more difficult investigations (physiology, society, government, &c.) consists in the applicability not of its doctrines, but of its method. Mathematics will ever remain the most perfect type of the Deductive Method in general; and the applications of Mathematics to the simpler branches of physics, furnish the only school in which philosophers can effectually learn the most difficult and important portion of their art, the employment of the laws of simpler phenomena for explaining and predicting those of the more complex. These grounds are quite sufficient for deeming mathematical training an indispensable basis of real scientific education, and regarding, with Plato, one who is ἀγεωμέτρητος, as wanting in one of the most essential qualifications for the successful cultivation of the higher branches of philosophy."

In addition to these authorities it may be remarked, that the new Regulations which were confirmed by a Grace of the Senate on the 11th of May, 1846, assign to Geometry and to Geometrical methods, a more important place in the Examinations both for Honors and for the Ordinary Degree in this University.

TRINITY COLLEGE,   R. P.
March 1, 1850.

The supplement to the School Euclid (about forty-eight pages) has been incorporated with this impression of the Fifth Edition.

TRINITY COLLEGE,
October, 1863.

EUCLID'S
ELEMENTS OF GEOMETRY.

BOOK I.

DEFINITIONS.

I.
A POINT is that which has no parts, or which has no magnitude.

II.
A line is length without breadth.

III.
The extremities of a line are points.

IV.
A straight line is that which lies evenly between its extreme points.

V.
A superficies is that which has only length and breadth.

VI.
The extremities of a superficies are lines.

VII.
A plane superficies is that in which any two points being taken, the straight line between them lies wholly in that superficies.

VIII.
A plane angle is the inclination of two lines to each other in a plane, which meet together, but are not in the same direction.

IX.
A plane rectilineal angle is the inclination of two straight lines to one another, which meet together, but are not in the same straight line.
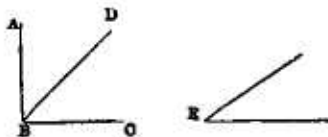
B

N.B. If there be only one angle at a point, it may be expressed by
a letter placed at that point, as the angle at E : but when several angles
are at one point B, either of them is expressed by three letters, of which
the letter that is at the vertex of the angle, that is, at the point in which
the straight lines that contain the angle meet one another, is put between
the other two letters, and one of these two is somewhere upon one of
these straight lines, and the other upon the other line.   Thus the angle
which is contained by the straight lines AB, CB, is named the angle
ABC, or CBA ; that which is contained by AB, DB, is named the angle
ABD, or DBA ; and that which is contained by DB, CB, is called the
angle DBC, or CBD.

X.

When a straight line standing on another straight line, makes the
adjacent angles equal to one another, each of these angles is called a
right angle ; and the straight line which stands on the other is called
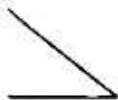a perpendicular to it.

XI.

An obtuse angle is that which is greater than a right angle.

XII.

An acute angle is that which is less than a right angle.

XIII.

A term or boundary is the extremity of any thing.

XIV.

A figure is that which is enclosed by one or more boundaries.