

**LECTURES ON THE  
PHILOSOPHY  
OF MATHEMATICS**

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Lectures on the Philosophy of Mathematics by James Byrnie Shaw

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**JAMES BYRNIE SHAW**

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Mathematics**

# Lectures on the Philosophy of Mathematics

*By*  
JAMES BYRNIE SHAW

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## PREFACE

In the spring of 1915 the author was invited to deliver a course of lectures before a club of graduate students of the University of Illinois on the subject "The Philosophy of Mathematics." This club consisted of students who had had an ordinary college course in mathematics, for the most part. This fact tended to narrow the field to be covered, inasmuch as the more difficult questions of mathematical philosophy had to be omitted. It tended to widen the field in the way of making it intelligible to all students of fair mathematical knowledge, which could be accomplished best by considering mathematics constantly in its historical development. This class of readers is the one directly addressed in the lectures. The large class of secondary and collegiate teachers of mathematics is also addressed to a great extent.

The author cherishes the hope that the professional philosopher too may find some interest in these lectures, even though the more delicate problems are omitted or only referred to. If the student of philosophy finds enough mathematics here to characterize the field and give him a broad view over its hills and valleys, he will see it from the mathematician's point of view. Many have already accomplished this, notably, in America, Royce, and in France a whole school, as E. Boutroux, Brunschvicg, Milhaud, LeRoy, Winter, Dufumier, not to mention the philosopher mathematicians, Poincaré, P. Tannery, J. Tannery, Picard, Borel, P. Boutroux, and others. This view is necessary adequately to account for mathematics.

The object of the lectures is to consider the whole field of mathematics in a general way, so as to arrive at a clear understanding of exactly what mathematics undertakes to do and how far it accomplishes its purpose; to ascertain upon what presuppositions, if any, which are extra-mathematical, the mathematician depends. The references at the ends of the chapters will enable the student who desires to go into the topics treated farther than the discussions of the text permit, to make a start at least on such reading. They are not intended to be exhaustive, but merely suggestive. Students should consult constantly the *Encyclopédie des sciences mathématiques*, the *Pascal Repertorium*, and the *Taschenbuch* of Teubner.

The author has gathered his material from many sources, to all of which he acknowledges his indebtedness. The original source where possible is given for all quotations in order that the reader may find the original setting. It is hoped that notions due to other mathematicians have been in every case exactly expressed. Critics of the relative importance attached herein to many developments of mathematics are asked to keep in mind the purpose of the lectures. The synoptical table (pp. 196-97) is given as a suggestive guide to the text, and is doubtless incomplete in many ways.

JAMES BYRNIE SHAW



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## CHAPTER I

### MEANING OF THE PHILOSOPHY OF MATHEMATICS

When the thinking person of the present day stops to reflect upon the facts of the wireless telegraph and the long-distance telephone, not to mention many other just as important marks of human progress, and remembers in his thinking that the existence of the wireless telegraph is due to deductions of Maxwell by means of theorems that depend upon the square root of minus one, and that the possibility of the long-distance telephone depends upon investigations of Pupin by means of theorems that depend more directly upon the modern theory of expansions in fundamental functions, he appreciates to the full the power of this branch of human learning. When he further learns that the existence of conical refraction was pointed out to the physicist by a mathematician before it was discovered in a laboratory; that the existence of Neptune was pointed out to the astronomer before his telescope had noticed this wanderer in the remote heavens; when he learns that the mathematician by a theory related to the solution of the problem of finding the roots of an algebraic equation is able to say to the mineralogist "you will never find more than thirty-two distinct types of crystals"—when he meets such facts as these, he must invariably ask: "Who is this magician whose wand creates the marvelous and whose penetrating eye searches the hidden corners of the universe?" He may still listen with an amused smile to the curious properties of four-dimensional space, may delight in the

escape from the weary wastes of infinite space in a Riemannian finite universe, may be bewildered by the Minkowski imaginary-time axis, may exhaust his imagination in the vain effort to draw the crinkly curves, but he will not look upon all these as vagaries of a mystical dreamer. He will rather desire to seek permission of this Queen to enter her realm and explore it. "Conterminous" with space and coeval with time is the kingdom of Mathematics; within this range her dominion is supreme; otherwise than according to her order nothing can exist; in contradiction to her laws nothing takes place. On her mysterious scroll is to be found written for those who can read it that which has been, that which is, and that which is to come." He will wish to know what things belong thereto, the content of this division of human knowledge whose riches grow steadily richer year by year, while scientific theories are mined, assayed, and worked to exhaustion; while social orders are laboriously built up, serve their usefulness, and break down into ruins—this, the only permanent organization constructed by the human race. Indeed, he has for more than twenty-five centuries endeavored to account for this Antaeus, who acquires new strength whenever he touches the earth, yet whose towering form disappears to the average eye, in the shining clouds. The philosophers of the race have designed systems that charmed the mind for a while, resting their foundations upon the verities of mathematics; but they have turned out to be only temporary structures, occupying but a small part of the foundations of this ever-expanding temple of learning, which has become more solid while they have crumbled to pieces.

<sup>1</sup> Spottiswoode, *Report of the British Association for the Advancement of Science*, 1878, p. 31.