MATHEMATICAL MONOGRAPHS. HARMONIC FUNCTIONS. NO. 5

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649345649

Mathematical monographs. Harmonic Functions. No. 5 by William E. Byerly

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

WILLIAM E. BYERLY

MATHEMATICAL MONOGRAPHS. HARMONIC FUNCTIONS. NO. 5

Trieste

MATHEMATICAL MONOGRAPHS.

EDITED BY

MANSFIELD MERRIMAN AND ROBERT S. WOODWARD.

No. 5.

HARMONIC FUNCTIONS.

BY

WILLIAM E. BYERLY,

PROFESSOR OF MATHEMATICS IN HARVARD UNIVERSITY.

FOURTH EDITION, ENLARGED.

FIRST THOUSAND.

NEW YORK:

JOHN WILEY & SONS.

LONDON: CHAPMAN & HALL, LIMITED.

1906.

First Edition, September, 1896.
Second Edition, January, 1898.
Third Edition, August, 1900.
Fourth Edition, January, 1906.

ROBERT DRUMMOND, PRINTER, NEW YORK.

EDITORS' PREFACE.

THE volume called Higher Mathematics, the first edition of which was published in 1896, contained eleven chapters by eleven authors, each chapter being independent of the others, but all supposing the reader to have at least a mathematical training equivalent to that given in classical and engineering colleges. The publication of that volume is now discontinued and the chapters are issued in separate form. In these reissues it will generally be found that the monographs are enlarged by additional articles or appendices which either amplify the former presentation or record recent advances. This plan of publication has been arranged in order to meet the demand of teachers and the convenience of classes, but it is also thought that it may prove advantageous to readers in special lines of mathematical literature.

It is the intention of the publishers and editors to add other monographs to the series from time to time, if the call for the same seems to warrant it. Among the topics which are under consideration are those of elliptic functions, the theory of numbers, the group theory, the calculus of variations, and non-Euclidean geometry; possibly also monographs on branches of astronomy, mechanics, and mathematical physics may be included. It is the hope of the editors that this form of publication may tend to promote mathematical study and research over a wider field than that which the former volume has occupied.

December, 1905.

AUTHOR'S PREFACE.

THIS brief sketch of the Harmonic Functions and their use in Mathematical Physics was written as a chapter of Merriman and Woodward's Higher Mathematics. It was intended to give enough in the way of introduction and illustration to serve as a useful part of the equipment of the general mathematical student, and at the same time to point out to one specially interested in the subject the way to carry on his study and reading toward a broad and detailed knowledge of its more difficult portions.

Fourier's Series, Zonal Harmonics, and Bessel's Functions of the order zero are treated at considerable length, with the intention of enabling the reader to use them in actual work in physical problems, and to this end several valuable numerical tables are included in the text.

CAMBRIDGE, MASS., December, 1905

CONTENTS.

HARMONIC FUNCTIONS.

ART. 1. HISTORY AND DESCRIPTION.

What is known as the Harmonic Analysis owed its origin and development to the study of concrete problems in various branches of Mathematical Physics, which however all involved the treatment of partial differential equations of the same general form.

The use of Trigonometric Series was first suggested by Daniel Bernouilli in 1753 in his researches on the musical vibrations of stretched elastic strings, although Bessel's Functions had been already (1732) employed by him and by Euler in dealing with the vibrations of a heavy string suspended from one end; and Zonal and Spherical Harmonics were introduced by Legendre and Laplace in 1782 in dealing with the attraction of solids of revolution.

The analysis was greatly advanced by Fourier in 1812–1824 in his remarkable work on the Conduction of Heat, and important additions have been made by Lamé (1839) and by a host of modern investigators.

The differential equations treated in the problems which have just been enumerated are

$$\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2} \qquad (1)$$