ELEMENTS OF GEOMETRY WITH NOTES

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OF

GEOMETRY,

WITH

NOTES.

BY J. R. YOUNG,

AUTHOR OF AN ELEMENTARY TREATISE ON ALGEBRA.

REVISED AND CORRECTED,

WITH ADDITIONS,

BY M. FLOY, JUN. A. B.

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ADVERTISEMENT.

The preparation of the present edition has been undertaken from a conviction that the work is more full and correct, at the same time better adapted to the use of learners than any that has preceded it.

The editions of Euclid by Simson and Playfair are works of inestimable value, both on account of the accuracy of the text, and the sound criticism displayed in the notes. They are not, however, well calculated for instruction; and although Playfair endeavoured to remedy this defect, (if so it may be called,) still he does not appear to have succeeded, having ventured but in a few instances to depart from the method laid down by Euclid. Excepting the above works, I know of none in the English language, however well adapted they may be for instruction, that are not deficient in some respects; it would be invidious to particularize,—suffice it to say that in one respect—the doctrine of proportion, they are all deficient: as this is an important part of the subject, a few words respecting it, may not be unnecessary.

Geometry, as is well known, treats of magnitudes that are often found to be incommensurable, that is, without a common measure; authors, generally, seem to lose sight of this, and in applying demonstrations to propositions, often for the purpose of simplifying them, break through the distinction which ought to be preserved. In the present work, the subject of proportion is managed in a very able and satisfactory manner; while it is as plain, perhaps, as can be expected, when it is considered that the demonstrations apply to magnitudes of all kinds; at the same time, the consideration of Euclid's definition of proportion, which is perplexing, and has caused much dispute among geometers, is entirely avoided.

Throughout the work, much industry, and research have been displayed by the author, particularly in the Seventh Book, on the properties of polygons; and indeed what has been said on the *converse* of the propositions in all the books, leaves little room for any further additions.

The arrangement, in some few instances, might have been altered for the better, as in Books 4th and 8th, and Books 6th and 7th; it may be thought also, that some of the problems in the 6th and 7th Books, might with propriety have been transferred to the 8th, which is devoted exclusively to the construction of problems. These are not, however, matters of much moment. The editor, as will be seen, has interspersed throughout the Books a choice selection of elegant and useful propositions. These are distinguished from those of the author, by the letters of the alphabet, and thus any interference with the author's chain of reasoning is prevented. It is to Bland's Geometrical problems, Legendre, Leslie, &c., that the editor is chiefly indebted for these; but it will be perceived that the demonstrations have been altered in many instances, and are given more in accordance with the spirit of the author.

Some additional matter has been given on the rectification and quadrature of the circle;—problems, which as Playfair justly observes, are often omitted in works on geometry, without good reasons; since the mensuration of the circle certainly belongs to the elements of the science. The author has given but one method,—that of Mr. James Gregory, for determining the *quadrature* of the circle. It is well known, however, that the quadrature is easily found, when the circumference is determined; and therefore it was thought proper to give methods for the *rectification* of the circle, as this is, doubtless, an easier problem than the other. Two methods of approximation for accomplishing this, will be found at the end of Book VII;—one by the continual bisection of an arc, the other by the trisection of the same; in the latter of which Mr. Young's method of solving Cubic equations, as given in his Algebra, is evidently employed with much success.

The notes interspersed throughout the work are numerous; it were tedious here to particularize; these may not prove uninteresting or useless to the student.

New York, March 1, 1833.

PREFACE.

Elements of geometry are by no means numerous in this country, a circumstance to be attributed to the almost universal preference given to Euclid; not, indeed, because the elements of Euclid is a faultless performance, but because its blemishes are so inconsiderable when compared with its extraordinary merits, that to reach higher perfection in this department of science has been generally supposed to be scarcely within the bounds of possibility, an opinion which the fruitless efforts of succeeding geometers to establish a better system have in a great measure confirmed. The superiority of Euclid's performance consists chiefly in the rigorous and satisfactory manner in which he establishes all his assertions, preferring in every case the most elaborate reasoning rather than weaken the evidence of his conclusions by the introduction of the smallest assumption.

On the continent, however, this high opinion of Euclid does not appear to prevail, and the rigour and elegance of his demonstrations, seem to be less appreciated. In all the modern French treatises on geometry, it is easy to discover a wide departure from that rigorous and accurate mode of reasoning so conspicuous in the writings of the ancient geometer. From this imputation even the celebrated *Elémens de Géométrie* of *Legendre*, "the first geometer in Europe," is not exempt, notwithstanding the masterly manner in which he has treated certain difficult parts of the subject. The greatest difficulty, however, in the whole compass of geometry is doubtless the doctrine of geometrical proportion. The manner in which Euclid establishes this doctrine is remarkable for the same rigour of proof that manifests itself throughout the other parts of his work, although it is universally acknowledged that from the difficulty of the subject his reasoning is so subtle and intricate, that to beginners it opposes a very serious obstacle. The grand aim, therefore, of geometers has been to deliver this part of Euclid's performance from its peculiar difficulties, without destroying the rigour and universality of his conclusions. All attempts to accomplish this important object have been unsuccessful; and those who have abandoned Euclid's method, and have treated the subject in a more concise and easy way, have greatly fallen short of that accuracy of reasoning so essential to geometrical investigations, and have arrived at conclusions that are not indisputably established, but only approximately true:—such is the doctrine of proportion as treated by geometers of the present day. It appears, therefore, that notwithstanding the recent translation of Legendre's celebrated work into our own language—the unqualified praise which has been bestowed upon it, and its extensive circulation throughout Europe, there are still blemishes to be removed and defects to be supplied; for, extraordinary as it may appear, Legendre has not, unfortunately, exercised his powerful talents upon the doctrine of proportion, but has entirely excluded the consideration of it from his elements, referring the student for requisite information "to the common treatises on arithmetic and algebra."* Now books on arithmetic and algebra can unfold the properties of proportion only as regards *numbers*, and numbers cannot extend to all classes of geometrical magnitudes, for some when compared are found to be incommensurable. The doctrine of proportion, therefore, in reference to these latter, cannot be rigorously inferred from any thing that may be established with regard to numbers or commensurable magnitudes.

Having adverted to these defects it remains for me now to give some brief account of the present attempt, and to state wherein I have endeavoured to render it more particularly worthy of examination.

And first it may be remarked, in reference to the general plan of the work, that I have taken a more enlarged and comprehensive view of the elements of geometry than I believe has hitherto been done, as I have paid particular attention to the *converse* of every proposition throughout these elements, having demonstrated the converse wherever such demonstration was possible, and in other cases shown that it necessarily failed. There can be no doubt that this comprehensive mode of proceeding, embracing as it does every thing connected with the subject, must afford the student entire satisfaction, and must also increase the accuracy, as well as the extent, of his geometrical knowledge; since he not only learns that under certain conditions a certain property must have place, but also whether or not it is possible for the same property to exist under any change of those conditions. The first, and I believe the only work in which converse propositions are fully considered, is that of *M. Garnier* entitled *Réciproques de la Géométrie*, and which, it appears, was intended to accompany the geometry of Legendre. In the present performance I have in several instances availed myself of this work of Garnier, although, in many other cases, I have found it expedient to adopt a different course. The only book on geometry with which I am acquainted, where the converse accompany the direct propositions to any extent, is the *Elémens de Géométrie par M. Develey*, a very comprehensive

* Dr. Brewster's translation of Legendre, page 48.

performance; but in many instances the converse propositions are not noticed, and in but very few cases is their failure shown to take place. This plan, therefore, is not systematic and uniform.

With regard to other, and more particular improvements, introduced into this work, may be noticed, proposition XIII. of the first book, taken, with little alteration, from the *Principes Mathématiques* of *M. da Cunha*, and which, as Professor Playfair remarked, in the Edinburgh Review, Vol. XX., is a decided improvement in elementary geometry, as it dispenses with an awkward subsidiary proposition of Euclid.

Upon the doctrine of proportion, which constitutes the fifth book of these elements, I have bestowed much labour and attention, and have, I hope, in some degree succeeded in diminishing the difficulties hitherto attendant upon that important subject.

The notes appended to this first part may, I think, be consulted by the student with advantage. I have therein endeavoured to point out some remarkable errors and inconsistencies into which modern geometers have fallen, particularly in reference to the theory of parallel lines, and the doctrine of proportion; and I believe many of these errors have hitherto remained unnoticed. A singular instance of this is shown in the notes to the sixth book, where a proposition in *Simpson's Geometry*, which has been for upwards of seventy years received as genuine, and adopted by more modern geometers, is proved to be false! Other instances of incautious reasoning are adduced from *Legendre*, *Dr. Simson*, and others, which it is doubtless of importance to detect and point out to the student, as indisputable proofs of the great caution necessary in geometrical reasoning.

Throughout the whole I have earnestly endeavoured to render this performance suitable to the wants of the student, and deserving of the approbation of the geometer. I can truly say that its composition has been attended with a great sacrifice, both of labour and expense: and its progress has been frequently interrupted by opposing circumstances. But if, notwithstanding, I shall have succeeded in rendering it worthy of notice, I shall consider myself fully recompensed for the pains it has cost me, and shall feel encouraged to proceed with more confidence and ardour in the remaining part of the subject.

J. R. YOUNG.

June 1, 1827.

The second part will contain the Geometry of Planes and Solids, with notes and an appendix on the Symmetrical Polyedrons of Legendre.