THE DIFFICULTIES OF ELEMENTARY GEOMETRY: ESPECIALLY THOSE WHICH CONCERN THE STRAIGHT LINE, THE PLANE, AND THE THEORY OF PARALLELS

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The Difficulties of Elementary Geometry: Especially Those Which Concern the Straight Line, the Plane, and the Theory of Parallels by Francis William Newman

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FRANCIS WILLIAM NEWMAN

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DIFFICULTIES

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ELEMENTARY GEOMETRY.

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THE STRAIGHT LINE, THE PLANE, AND THE THEORY OF PARALLELS.

BY

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The reader is requested, on reading Art. 123, to peruse Appendix II., and at Artt. 55, 150, to refer to the Note at the end of the Volume.

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