

**THE ELEMENTARY
PROPERTIES OF THE
ELLIPTIC FUNCTIONS WITH
EXAMPLES**

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The Elementary Properties of the Elliptic Functions with Examples by Alfred Cardew Dixon

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ALFRED CARDEW DIXON

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ELLIPTIC FUNCTIONS

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BY

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PREFACE.

THE object of this work is to supply the wants of those students who, for reasons connected with examinations or otherwise, wish to have a knowledge of "the elements of Elliptic Functions, not including the Theory of Transformations and the Theta Functions." It is right that I should acknowledge my obligations to the treatise of Professor Cayley and to the lectures of Dr. Glaisher, as well as to the authorities referred to from time to time. I am also greatly indebted to my brother, Mr. A. L. Dixon, Fellow of Merton College, Oxford, for his kind help in reading all the proofs and working through the examples, as also for his valuable suggestions.

A. C. DIXON.

DUBLIN, October, 1894.

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ELLIPTIC FUNCTIONS.

CHAPTER I.

INTRODUCTION. DEFINITION OF ELLIPTIC FUNCTIONS.

§1. In the earlier branches of mathematics functions are defined in various ways. Some are the results of the fundamental operations of algebra. $x+1$, $2x$, x^2 are such functions of x . Others are introduced by the inversion of those operations; such are $x-1$, $1/x$, \sqrt{x} ; and others by conventional extensions of them, as x^{-1} , e^x . It is not easy to draw the line of distinction between the two last-named classes. Sometimes, again, geometrical constructions are used in the definition, as in the case of the trigonometrical functions.

§2. The elliptic functions cannot readily be defined in any of the foregoing ways; their fundamental property is that their differential coefficients can be expressed in a certain form, and as this is a somewhat new way of defining a function, we shall take one or

D. E. F.

A.