AN ELEMENTARY TREATISE ON SPHERICAL HARMONICS AND SUBJECTS CONNECTED WITH THEM

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An Elementary Treatise on Spherical Harmonics and Subjects Connected with Them by N. M. Ferrers

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N. M. FERRERS

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SPHERICAL HARMONICS.

AN ELEMENTARY TREATISE

ON

SPHERICAL HARMONICS

AND SUBJECTS CONNECTED WITH THEM.

BY THE

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