

**AN ELEMENTARY TREATISE
ON SPHERICAL HARMONICS
AND SUBJECTS CONNECTED
WITH THEM**

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An Elementary Treatise on Spherical Harmonics and Subjects Connected with Them by N. M. Ferrers

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N. M. FERRERS

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SPHERICAL HARMONICS.

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BY THE

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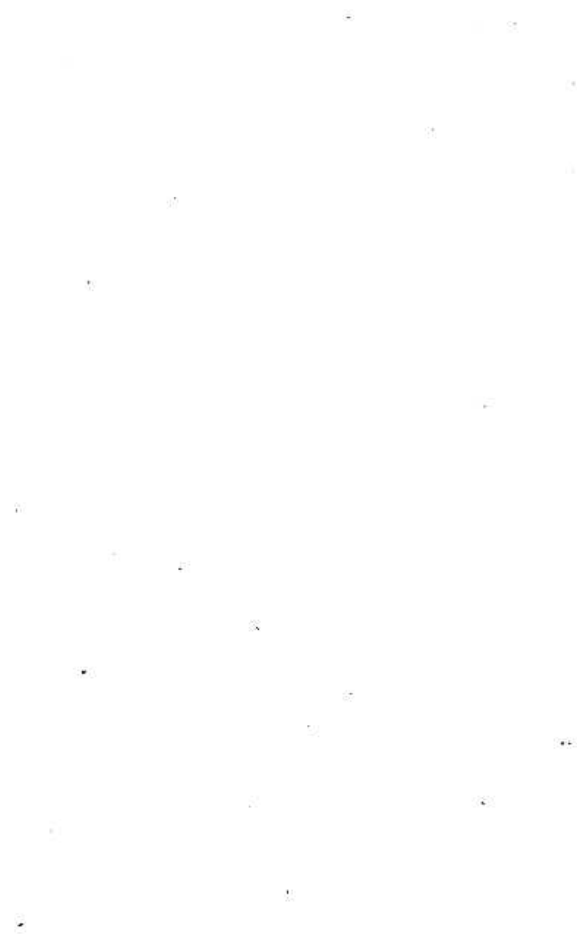


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