THE THEORY OF ERRORS AND METHOD OF LEAST SQUARES

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BY

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PREFACE.

THE basis adopted in this book for the theory of accidental errors is that laid down by Gauss in the *Theoria Motus Corporum Cœlestium* (republished as vol. vii of the *Werke*), which may be described for the most part in his own words, as follows :

"The hypothesis is in fact wont to be considered as an axiom that, if any quantity has been determined by several direct observations, made under similar circumstances and with equal care, the arithmetical mean between all the observed values presents the most probable value, if not with absolute rigor, at least very nearly so, so that it is always safest to adhere to it." (Art. 177.)

Then introducing the notion of a law of facility of error to give precise meaning to the phrase "most probable value," we cannot do better than to adopt that law of facility in accordance with which the arithmetical mean is the most probable value. After deriving this law and showing that it leads to the principle of least squares, he says : "This principle, which in all applications of mathematics to natural philosophy admits of very frequent use, ought everywhere to hold good as an axiom by the same right as that by which the arithmetical mean between several observed values of the same quantity is adopted as the most probable value." (Art. 179.)

Accordingly no attempt has been made to demonstrate the principle of the arithmetical mean, nor to establish the exponential law of facility by any independent method. It has been deemed important, however, to show the self-consistent nature of the law, in the fact that its assumption for the errors of direct observation involves as a consequence a law of the same form for any linear function of observed quantities, and particularly for the final determination which results from our method. This persistence in the form of the law has too frequently been assumed, in order to simplify the demonstrations; but at the expense of soundness.

No place has been given to the so-called criteria for the rejection of doubtful observations. Any doubt which attaches to an observation on account of the circumstances under which it is made, is recognized, in the practice of skilled observers, in its rejection, or in assigning it a small weight at the time it is made; but these criteria profess to justify the subsequent rejection of an observation on the ground that its residual is found to exceed a certain limit. With respect to this Professor Asaph Hall says: "When observations have been honestly made I dislike to enter upon the process of culling them. By rejecting the large residuals the work is made to appear more accurate than it really is, and thus we fail to get the right estimate of its quality." (*The Orbit of Iapetus*, p. 40, *Washington Observations for* 1882, *Appendix I.*)

The notion that we are entitled to reject an observation, that is, to give it no weight, when its residual exceeds a certain limit, would seem to imply that we ought to give less than the usual weight to those observations whose residuals fall just short of this limit, in fact that we ought to revise the observations, assigning weights which diminish as the residuals increase. Such a process might appear at first sight plausible, but it would be equivalent to a complete departure from the principle of the arithmetical mean and the adoption of a new law of facility. For this we have no justification, either from theory or from the examination of the errors of extended sets of observations.

In the discussion of Gauss's method of solving the normal equations, the notion of the 'reduced observation equations' (see Arts. 154, 155) which gives a new interpretation to the 'reduced normal equations' has been introduced with advantage. This conception, although implied in Gauss's elegant discussion of the sum of the squares of the errors (see Art. 160), seems not to have appeared explicitly in any treatise prior to the third edition of W. Jordan's *Handbuch der Vermessungskunde* (Stuttgart, 1888). To this very complete work, and to Oppolzer's *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*, I am indebted for the forms recommended for the computations connected with Gauss's method, and for many of the examples.

W. W. J.

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