DEVELOPMENTS OBTAINED BY CAUCHY'S THEOREM: WITH APPLICATIONS TO THE ELLIPTIC FUNCTIONS

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BY

HENRY P. MANNING

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PREFACE.

The basis of this paper is an "*Extrait d'une lettre de M. Gomes Teixeira à M. Hermite*," published in the *Bulletin des Sciences Mathematiques* for Sept. 1890. He showed how, by the employment of Cauchy's theorem, we can get an analytical representation of a function in the form of a series going according to ascending powers of the sine. I have given his method in §§3–5, and my treatment of the question of Convergence is essentially the same as his, although the details have been somewhat simplified.

By reducing to another form the function whose residue gives the representation sought, I have been able to deduce the law of the series; but the forms which presented themselves in this process suggested another and much simpler form of function to start with, and by this I have obtained a general form of development in powers of any holomorphic function and one or two interesting theorems concerning these functions. These general formulae have been applied to get developments in powers of the tangent and of the sn, and I have given some of the different forms which these developments may take and some examples of their application. Finally, the formulae which give developments in powers of the sine and of the sn have been employed to obtain these developments for sn (mx), cn (mx), and dn (mx). In this application I have made use of the method of "ternary paths" due to M. Desiré André and employed by him in calculating the developments of $\text{sn}^2 x$, $\text{cn}^2 x$, and $\text{dn}^p x$ by Maclaurin's formula* (*Annales de L'Ecole Normale Supérieure*, 2 *Série*, T. 6).

Dr. Craig first called my attention to the communication of M. Teixeira and suggested to me the investigation which has led to these results, and I have had the benefit of his advice in all of this work.

BALTIMORE, *May* 1, 1891.

* The use of the function $\phi(x)$ to represent the three elliptic functions and the method of treating the cn (mx) given in §36 were also suggested by André's paper.

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INTRODUCTION.

1. The possibility of the analytical representation of a function by means of a series, and the limits of convergence of such series, have been among the most difficult and important questions of analysis. And such representations are continually called for in the applications of mathematics, particularly to physical and astronomical problems. It was the question of a vibrating chord that gave rise to a form of development sought for by D'Alembert, Euler, Bernoulli, and Lagrange, but completely established first by Fourier in 1807.* There are many kinds of development more or less useful and interesting, but the difficulty of determining the limits of convergence and of applying a given form to a given case is often well-nigh insuperable. Many forms that have been given to the remainders, for example, of Taylor's and Maclaurin's formulae are difficult to test. It is only with the modern notion of the curvilinear integral and Cauchy's beautiful theorems that we have been able to arrive at a method of testing this matter which is easy and of wide application. Hermite† has shown how by means of the curvilinear integral we may obtain both of these formulae and at the same time simple criteria of their availability, and that, too, for the complex variable as well as the real variable. Finally, having these formulae and the notion of residue, we are able, by virtue of Cauchy's theorem, to obtain a great number of developments. This method has not been employed directly to obtain Fourier's series, but Dini, in his work already

* Ulisse Dini: *Serie di Fourier e Altre Representasioni Analitiche delle Funsione di Una Variabile Reale. Parte Prima*, §2.

† *Cour de M. Hermite, Quatrième édition, 9e Leçon.*