# MATHEMATICAL QUESTIONS WITH THEIR SOLUTIONS, FROM THE "EDUCATIONAL TIMES", VOL. XXIX

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Mathematical Questions with Their Solutions, from The "Educational Times", Vol. XXIX by W. J. C. Miller

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W. J. C. MILLER

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Trieste

# MATHEMATICAL QUESTIONS

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# SOLUTIONS,

FROM THE "EDUCATIONAL TIMES,"

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Papers and Solutions not published in the "Educational Cimes."

EDITED BY

W. J. C. MILLER, B.A., REDISTRAN OF THE SUBMERAL REDICAL COURCIL.

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#### CORRIGENDA.

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#### VOL. XXII.

Pages viil. and 27, for No. of Question 4350, read No. 4342

#### VOL XXVIII,

Page 68, last line, for  $(r^2z^2)$  read  $(r^2-z^2)$ . Page 69, line 7 from bottom, for  $(R^2-z^2)$  read  $(R^2-z^2)t$ Page 86, line 9 from bottom, for inside read inside.

#### VOL XXIX.

VOL XXIX.
Page 40, line 12 from bottom, for No. 138 read 134.
Puge 50, line 4 from bottom, after "values" insert "of."
Page 60, line 13 from bottoin, read 1."
Page 61, line 13 from bottoin, read "the respective probabilities of the promiscious occurrence of the two cases are 1 and 1."
Page 74, line 11, for No. 186 read 137.
Page 90, line 15, for 2, 8, 6 read 2, 3, 6.
Page 90, line 15, for No. 139 read 140.

N.B.—Of this suries twenty-nine volumes have now been published, each volume containing, in addition to the papers and solutions that have appeared in the *Educational Times*, about the same quantity of new articles, and comprising contributions, on all branches of Mathematics, from most of the leading Mathematicians in this and other comprising contributions. other countries.

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 Totor Strinity College. Dublin

### CONTENTS.

## Mathematical Papers, &c.

No.	eneral contraction and an and a second database	Page
131	Notes on Random Chords. By the Editor	17
182	Contraposition : By Alexander J. Ellis, F.R.S.	34
133	To find the Directrix of the Parabola $(sx + \delta y)^2 + 2dx + 2sy + f = 0$ . By W. Gallstly, B.A.	37
134	On the Random Chord Question. By Helen Thomson	40
135	Note on Mr. Woolhouse's Solution of his Question 5502. By Professor Monok, M.A.	
136	Miss Blackwood's Reply to Helan Thomson's Verses on "Random Chords."	62
187	Conic Constructions. By E. J. Lawrence, M.A	74
138	Note on Question 5458. By the Editor	94
139	On the Sign of any Term of a Determinant. By G. B. Dick, M.A.	99
140	Note on Professor Monck's Solution of Question 6602. By W. S. B. Woolhouse, F.R.A.S.	

### Solbed Questions.

4253.	(H. S. Monck, M.A.)-A series of Pythagorean triangles with	
	the difference between the hypothenuse and one side equal to	
	s, can always be obtained by beginning with the triangle 3s,	
	5n, 4n, and taking the upper figure as negative in each odd	
	term of the series given in Quest. 4102. Find in what cases a	
	distinct series with the same difference can be obtained	23
	지 않는 것은 것은 것은 것은 것을 가지 않는 것 같은 것은 것은 것은 것을 가지 않는 것을 것을 하는 것을 가지 않는 것을 하는 것을 하는 것을 수 있다.	12000

52

67 (Brofessor Cayley, F.R.S).—Given three conics passing through the same four points; and on the first a point A, on the second a point B, and on the third a point C. It is required to find on the first a point A', on the second a point B', and on the third a point C', such that the intersections of the lines A'B' and AC, A'C' and AB, lie on the first conic; B'C' and CB, C'B' and CA, lie on the third conic: 20

#### CONTENTS.

No.			Page
4902.		erifield, F.R.S.)—Can a sphere be touched by more e other equal spheres ?	85
5067.	(S. Tebay,	B.A.)—Let $x_1 + x_2 + \dots + x_n = 1$ , where	114
	a,	$> x_1 > \ldots > x_n$ ; find the mean value of $r_1^1$	25
5090.	(C. Leudes	dorf, M.A.)-Evaluate (1) the equation	
(a	= [(ad when ld +	$2fy + 2gx + 2hxy)(ax'^2 + by'^2 + c + 2fy' + 2gx' + 2hx'y') + hy' + g)x + (hx' + by' + f)y + (gx' + fy' + c)]^2,$ my' = 0, and x' and y' become infinite ; and (2) give rical interpretation.	
<b>5</b> 101.	of a sphere	, M.A.)—An auger-hole is made through the centre ; show that the average of the volume removed is,	
	1.1.199.142.948.4514	the volume of the sphere, $1 - \frac{3}{16}\pi$ .	
5111.	(Professor	Wolstenholme, M.A.) - 1. If $a, \beta$ be two angles	
	such that	$[1 + 2 (\cos \alpha)^{\frac{1}{2}}] [1 + 2 (\cos \beta)^{\frac{1}{2}}] = 3(A),$	
	prove that	$\frac{(1-3\cos^2\alpha)^4}{\sin^2\alpha\cos\alpha} = \frac{(1+3\cos^2\beta)^4}{\sin^3\beta\cos\beta}.$	

2. A circle and a rectangular hyperbola such passes through the centre of the other, and a.  $\beta$  are the two scute angles of intersection of the curves at their two real common points; prove that  $a, \beta$  will satisfy the equation ( $\Delta$ ), and that the squares of their latera recta are in the ratio

(1 + 8 cos<sup>4</sup> α)<sup>§</sup> : g sin<sup>3</sup> α cos α......(B).

(1+3 cost  $\alpha_i$ ):  $(3 \text{ min}^* \alpha \cos \alpha, \dots, (B))$ . 3. If a circle and a purabola he such that the circle passes through the focus of the parabola, and its centre lies on the directrix, prove that their angles of intersection satisfy the equation (A), and their latera rocts are in the ratio (B). 4. If a rectangular hyperbola and a parabola be such that the centre of the hyperbola is the focus of the parabola, and the directrix of the purabola touchas the hyperbols; then, if their acute angles of intersection be  $\pi-2\alpha$ ,  $\pi-2\beta$ , prove that  $\alpha$ ,  $\beta$ will satisfy the equation (A), and that the equates on the latera recta are in the ratio (B). recta are in the ratio (B). 31

- 5146. (S. Boberts, M.A.)—Given a pencil of rays and a system of con-centric circles; prove (1) that if one set of intersections range on a straight line, the other intersections lie on a circular cubic, having a double point at the origin of the pencil and the double focus at the common centre of the circles; and (2) determine therefrom, with reference to a system of parabolas having the same focus and axis, the locus of the points the normals at which intersect in a fixed point. ..... 56
- $\begin{array}{c} \delta 173. \quad (\textbf{H. T. Gerrans, B.A.}) & \textbf{Find the sums of the infinite series} \\ \frac{x^0}{2} + \frac{x^4}{5} + \frac{x^{12}}{8} + \frac{x^{13}}{11} + \frac{x^{14}}{14} + \textbf{\&c.}, \ \frac{x^4}{3} \frac{x^4}{3} + \frac{x^7}{5} \frac{x^4}{7.9} + \textbf{\&c.} \quad \textbf{39} \end{array}$
- 5192. (H. T. Gerrans, B.A.)—AB is a fixed diameter of a circle, OA a chord, ON an ordinate of the diameter, AP a line drawn so that  $\angle OAP = \angle OAN$ , and AP = AN; find the locus of P. ... 30
- 5212. (Professor Wolstenholms, M.A.)—A circle is drawn touching both branches of a fixed hyperbole in P, P', and meeting the asymptotes in L, L', M, M': prove that (1) LL'-MM'-major

viii

CONTENT	8.

N

	CONTENTS.	ix
No.	axis; (2) the tangents at L, M meet in one focus, and those at L', M' in the other, and the angle between either pair is constant, supplementary to the angle between the asymptotes; (3) the directrices bisect LM, L'M'; (4) P' bisects LL', M', L'M, L'M'; (5) the tangents at L, L' intersect on a reotangular hyperbola passing through the foci and having one of its asymptotes expendent M M' (bocause $\angle CSL + \angle CSL' = angle between the asymptotes; (6) LM, L'M' (touch parabolas having their foci at the foci of the hyperbola, and the tangents at their vertices the directrices of the hyperbola.$	Page 29
<b>5</b> 224.	(Rev. H. G. Day, M.A.)—On each of $n$ pillars, whose heights, in according order of magnitude, are $e_1, e_2, e_3, \ldots, e_n$ , points are taken at random; find the chance of the point so taken on the rth pillar being the highest.	34
5268.	(E. B. Seitz.)—Two equal circles, each of radius $r$ , are drawn on the surface of a circle of radius $2r$ ; show that the	
	average area common to the two circles is $\left(1-\frac{16}{3\pi^2}\right)\pi r^2$	84
5299.	(L. H. Rosenthal.)—Solve the simultaneous equations, $a^2 - aa^2 + (b-2y)x + ay - s = 0$ (1),	
¢304.	$x^2y-axy-(y^2-by+d)=0$ (2). (Professor Clifford, F.R.S.)—Prove that the negative pedal of an ellipse, in regard to the centre, has an easys and four nodes; find their positions, and the length of the are external to the ellipse between two real cusps; and account fully for the apparent reduction of the corve to a circle and two parabolas	70
<b>6</b> 315.	respectively, in special cases	47
	is less than the side of the only is $\frac{13}{8\pi}$	111
<b>5</b> 820.	(J. J. Walker, M.A.)—If normals to the ellipse $b^{5}x^{2} + a^{3}y^{2} - a^{2}b^{2} = 0$ be drawn from any point on the curvo $(a^{2}x^{2} + b^{2}y^{2} - a^{4})^{2} + 54a^{2}b^{2}c^{4}x^{2}y^{2} = 0,$	
5331.	prove that they form an harmonic pencil	38
	$27 ay^2 = (3a - x) (x + 6a)^2$ ; (3) the normal of the pedal exceeds the ordinate by a fixed length; (4) the arc measured from the vertex to any point is equal to the intercept of the normal on the axis of y; and (3), if a heavy uniform claim be ited tightly iound a curve, such that the pressure per unit is equal to tho weight of a unit of length of the chain, this curve must be the first negative focal pedal of a parabola.	27
5339.	(Hugh McColl, B.A.) — In the quadratic equation $x\theta^2 + x\theta + y = 0$ , the coefficient $x$ is taken at random between 0 and 3, the coefficient $y$ between $-1$ and 4, and the coefficient $s$ between $-3$ and 3; show that the chance that the following $\delta$	