NEW ELEMENTS OF GEOMETRY

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OF

GEOMETRY.

BY SEBA SMITH.

"GOD SAID, LET THERE BE LIGHT; AND THERE WAS LIGHT."

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1850.

PART FIRST.

THE PHILOSOPHY OF GEOMETRY.

SECTION I.

IMPORTANCE OF THE SCIENCE, AND ITS DIFFICULTIES.

"THE invention of forms," says Lord Bacon, the great founder of inductive philosophy, "is of all other parts of knowledge the worthiest to be sought, if it be possible to be found." And in the same connection he adds: "As for the possibility, they are ill-discoverers that think there is no land when they can see nothing but sea."

Plato also regarded *forms* as the true object of knowledge; but in the judgment of Bacon he "lost the real fruit of his opinion, by considering of forms as absolutely *abstracted from matter*;" by which means he was led into theological speculations, "wherewith all his natural philosophy is infected."

In the opinion of Pythagoras, the study of the mathematics, including geometry, was "the first step toward wisdom." The pupils in his school first became mathematicians; and after they had made sufficient progress in geometrical science, they were conducted to the study of nature, the investigation of

primary principles, and the consideration of the attributes of Deity.

Plato arrived at such a reverence for geometry, that he had inscribed over the door of his academy where he taught philosophy, "Let no one who is ignorant of geometry enter here." And when his opinion was asked concerning the probable employment of Deity, he is said to have replied, "He geometrizes continually;" by which he undoubtedly meant that the great Author of nature established and governs the universe by geometrical laws.

Also the learned and pious Dr. Barrow held geometry in such estimation, that he considered the contemplation of it as not unworthy of the Deity; and in publishing an edition of the works of Apollonius, he inscribed it with the words, "God himself geometrizes. O Lord, how great a geometer art thou!"

And in testimony of the truth and immutability of the principles of geometry, Aristotle, the great master of ancient philosophies, declared that "the poles of the world will be sooner removed out of their places, and the fabric of nature destroyed, than the foundations of geometry fail, or its conclusions be convinced of falsity."

And yet this first and most important of the sciences—most important, because lying at the foundation of all other sciences—so clear in its principles, so certain in its conclusions, so venerable for its antiquity, hoary with the lapse of thousands of years, and honored in every age by the earnest investigations of the master-minds of men—this grand fabric of geometry, so beautiful in its proportions, and so magnifi-

cent in extent, rests in part on a false foundation. One of the corner-stones upon which it was first erected was given with false dimensions, and must be removed, and its place supplied by the true corner-stone, before the structure can be made perfect throughout, and present an unbroken harmony in all its relations.

To remove that false corner-stone and supply the true, is the object intended by the present treatise. Should I be met at the threshold by the incredulous world, and reproachfully or satirically asked, in the words of Paul, "Who is sufficient for these things?" I shall reply only in the humble spirit of the same apostle, when he declared that "God hath chosen the foolish things of the world to confound the wise; and God hath chosen the weak things of the world to confound the things which are mighty."

The error, which I allege to exist in the fundamental principles of geometry, is embodied in the first definitions of the science, as given by Euclid, and as adopted and followed in the many hundreds of works written on the subject for the last two thousand years. Euclid, and I believe all other geometers who have written hitherto, take their stand upon these definitions, viz.:

"A line is length without breadth;" and "a surface is length and breadth without thickness."

I meet these definitions at once, and declare that every mathematical line has a *breadth*, as definite, as measurable, and as clearly demonstrable, as its length; and that every mathematical surface has a *thickness* as

definite, as measurable, and as clearly demonstrable, as its length or breadth.

Will it be answered here, that the demonstrations given by geometers are clearly and unquestionably true, and therefore, if there be an error in one or two of their definitions or assumed principles it affects not their conclusions, and must be a matter of but little consequence? Though such an answer could scarcely be expected to come from a mind imbued with sound principles of philosophy, it may still be worth while to dwell upon it for a moment. Astronomers got along very well before Galileo's time, upon the hypothesis that the earth was the center of the solar system. They calculated eclipses on that hypothesis, and the eclipses came out right, and verified their calculations. Was it, therefore, a matter of but little consequence, that their whole system was based on a false foundation? The conclusions which they could reach from that foundation were clearly and unquestionably true; but there are many vast and important truths in the science of astronomy, which they never could have reached till their fundamental error was discovered, and the earth allowed to revolve about the sun.

So there are many important truths in geometry demonstrated in this work, and many more doubtless yet to be demonstrated, which never could have been reached till the true nature of lines and surfaces was discovered and their proper quantities demonstrated.

It is certainly unphilosophical to admit that a truth, lying at the foundation of any science, can be unim-

portant. Professor Playfair has well and forcibly said: "The truths of geometry are all necessarily connected with one another, and the system of such truths can never be rightly explained, unless that connection be accurately traced wherever it exists. It is upon this, that the beauty and peculiar excellence of the mathematical sciences depend. It is this, which, by preventing any one truth from being single and insulated, connects the different parts so firmly, that they must all stand, or fall together."

If it is a truth, therefore, that mathematical lines have a definite and measurable breadth, and that mathematical surfaces have a definite and measurable thickness, that truth must unquestionably be of great importance to the science of geometry and to all mathematics. And the want of a knowledge of this truth, I think, has hitherto prevented the true relation between numbers, magnitudes, and forms, from being clearly and properly understood. It is not strange therefore, that while a fundamental principle in mathematics remained shrouded in darkness, the professors of that science should have been led into a thousand laborious and useless speculations, upon questions in which that unknown principle was necessarily involved. Indeed from these causes, the mathematical sciences, like a very luxuriant vine left without pruning, have run out into immense quantities of foliage, bearing comparatively but little fruit. This state of things has become a reproach to mathematics. The writer of an able article in the Edinburgh Encyclopedia remarks, "The luxuriance of modern analytical speculation is arrived at such a point as to startle the most

industrious, and to render an equally perfect knowledge of all its parts, *no longer attainable by one individual.*" And a writer of equal ability in the London Encyclopedia pursues the subject in a vigorous and satirical vein as follows. "Let the mathematics be encouraged and patronized. Let them be cultivated to the fullest extent, even with considerable waste of mental power and loss of money, to discover the north-west passage in the polar regions of fluxionary creation; to find out some new calculus, whether differential or integral. Were there no probable, or even possible results, as to such a mixed, impure, vulgar entity, as utility, in contending with practicability, and penetrating to a high mathematical latitude of discovery; were it merely for the sake of the contention of mind, or to have it proved how far the algebraic analyst can go up out of sight in some new-invented infinitesimal balloon; in short if the mathematical progression were a thousand miles ahead of any practical purpose or advantage whatever, we would not be for terminating its career. We have spare hands enough, and spare heads too; and as all cannot find useful employment, it is better perhaps that they should be out of the way of idleness and mischief, by digging mathematical holes and filling them up again, or in perpetual motion to discover new methods of contention of mind, new calculuses, new analyses, new fluxions, new infinitesimals, to rival and supersede the old ones, ad infinitum. Only let us know, if possible, what the mathematics are about, and wherein their infinite quantity of excellence consists."