A TREATISE ON THE ELEMENTS OF ALGEBRA; PP. 7-227

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649067589

A Treatise on the Elements of Algebra; pp. 7-227 by B. Bridge

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

B. BRIDGE

A TREATISE ON THE ELEMENTS OF ALGEBRA; PP. 7-227

Trieste

A

TREATISE

ON

THE ELEMENTS

OF

ALGEBRA.

BY THE

REV. B. BRIDGE, B.D. F.R.S.

FELLOW OF ST. PETER'S COLLEGE, CAMBRIDGE.

FIFTH EDITION.

LONDON:

PRINTED BY R. WATTS:

AND SOLD BY T. CADELL, STRAND, LONDON;

DEIGHTONS, NICHOLSONS, AND BARRETT, CAMBRIDGE;

AND PARKER, OXFORD.

1821.

D.

CHAP. I.

ON THE

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF ALGEBRAIC QUANTITIES.

14. PREVIOUS to the application of the fundamental rules of Arithmetic to Algebraic quantities, it may be proper to observe, that the symbols + and − are distinguished from all the other signs or symbols, by giving a kind of *quality* or *affection* to the quantities to which they are annexed. As all those terms which have the sign + prefixed to them are to be *added*, and those which have the sign − prefixed to them are to be *subtracted*, from the terms which precede them, the former have a tendency to *increase*, and the latter to *diminish*, the quantities with which they are combined. A *compound* quantity, $x-a$ for instance, will therefore be positive or negative, according to the effect which it produces upon some third quantity c. Thus, if x be *greater* than a, the $c+x-a$ (since x is *added* and a *subtracted*) is greater than c; if x be *less* than a, then $c+x-a$ is less than c; *i.e.* "if x be *greater* than a, $x-a$ is positive; and if x be *less* than a, then $x-a$ is negative." In the same manner it might be shewn that the expression $a-b+c-d$ is positive or negative, according as $a+c$ is greater or less than $b+d$; and so of all compound quantities whatever.

ADDITION.

III.

ADDITION.

From the division of algebraic quantities into *positive* and *negative*, *like* and *unlike*, there arise three cases of Addition.

Case I.

To add like quantities with like signs.

15. In this case, the rule is "To add the coefficients of "the several quantities together, and to the result annex the "common sign, and the common letter or letters;" for it is evident, from the common principles of Arithmetic, if $+2a$, $+3a$, and $+5a$ be added together, their sum must be $+10a$; and if $-3b^2$, $-4b^2$, and $-8b^2$ be added together, their sum must be $-15b^2$.

Ex. 1.	Ex. 2.	Ex. 3.
$2x+3a-4b$	$7x^2+3xy-5bc$	$4a^3-3a^2+1$
$3x+2a-5b$	$9x^2+2xy-7bc$	$2a^3-a^2+17$
$4x+8a-7b$	$11x^2+5xy-4bc$	$5a^3-2a^2+4$
$9x+4a-6b$	$x^2+4xy-bc$	$3a^3-7a^2+3$
$5x+7a-9b$	$x^2+9xy-2bc$	a^3-a^2+10
$23x+24a-31b$	$29x^2+23xy-19bc$	$15a^3-14a^2+35$

Ex. 4.	Ex. 5.	Ex. 6.
$3x^3+4x^2-x$	$7a^3-3a^2b+2ab^2-3b^3$	$2x^2y-3x+2$
$2x^3+x^2-3x$	$4a^3-a^2b+ab^2-b^3$	$4x^2y-2x+1$
$7x^3+2x^2-2x$	$a^3-2a^2b+3ab^2-5b^3$	$3x^2y-5x+10$
$4x^3+x^2-x$	$5a^3-3a^2b+4ab^2-2b^3$	$x^2y-x+15$

In these Examples it may be observed that some of the quantities have *no coefficient*. In this case, *unity* or 1 *is always understood*. Thus, in adding up the *first* column of Ex. 2. we say, $1+1+11+9+7=29$; in the *third*, $2+1+4+7+5=19$; and so of the rest.

Case

Case II.

To add like quantities with unlike signs.

16. Since (by Art. 14) the compound quantity $a+b-c+d-e$ &c. is positive or negative, according as the sum of the positive terms is greater or less than the sum of the negative ones, the aggregate or sum of the quantities $2a-4a+7a-3a$ will be $+2a$, and of the quantities $7b^2-5b^2+2b^2-8b^2$ will be $-4b^2$; for in the former case, the excess of the sum of the positive terms above the negative ones is $2a$, and in the latter $4b^2$. Hence this general rule for the addition of like quantities with unlike signs, "Collect "the coefficients of the *positive* terms into one sum, and also "of the *negative*; subtract the *lesser* of these sums from "the *greater*; to this *difference*, annex the sign of the "*greater* together with the common letter or letters, and the "result will be the sum required."

If the aggregate of the positive terms be *equal* to that of the negative ones, then this *difference* is equal to 0; and consequently the sum of the quantities will be equal to 0, as in the *second* column of Ex. 2. following.

Ex. 1.	Ex. 2.	Ex. 3.
$4x^2-3x+4$	$-7ab+3bc-xy$	$-5x^3+13x^2$
$-2x^2+x-5$	$-ab+2bc+4xy$	$-2x^3-4x^2$
$3x^2-5x+1$	$3ab-bc+2xy$	$7x^3+x^2$
$7x^2+2x-4$	$-2ab+4bc-3xy$	$9x^3-14x^2$
$-x^2-4x+13$	$5ab-8bc+xy$	$-13x^3-2x^2$
$11x^2-9x+9$	$-2ab\ *\ +3xy$	$-4x^3-6x^2$

Ex. 4.	Ex. 5.	Ex. 6.
$4x^3-2x+3y$	$5a^2-2ab+b^2$	$4x^2y^2+2xy-3$
$-x^3+4x-y$	$-a^2+ab-2b^2$	$-x^2y^2-xy-1$
$7x^3-x+9y$	$4a^2-3ab+b^2$	$3x^2y^2+4xy-5$
$9x^3+21x-2y$	$2a^2+4ab-4b^2$	$-9x^2y^2-2xy+9$

Case III.

17. There now only remains the case where *unlike* quantities are to be added together, which must be done by collecting them together into one line, and annexing their proper signs; thus the sum of $3x$, $-2a$, $+5b$, $-4y$, is $3x-2a+5b-4y$; except when *like* and *unlike* quantities are mixed together, as in the following examples, where the expressions may be simplified, by collecting together such quantities as will coalesce into one sum.

Ex. 1.

$$\begin{array}{l} 3ab+x-y \\ 4c-2y+x \\ 5ab-3c+d \\ 4y+x^2-2y \\ \hline 8ab+2x-y+c+d+x^2 \end{array}$$

Collecting together *like* quantities, and beginning with $3ab$, we have $3ab+5ab=8ab$; $+x+x=+2x$; $-y-2y+4y-2y=-y$; $4c-3c=+c$; besides which there are the two quantities $+d$ and $+x^2$, which do not coalesce with any of the others; the sum required therefore is $8ab+2x-y+c+d+x^2$.

Ex. 2.

$$\begin{array}{l} 4x^2-2xy+1-3y+4x^3 \\ 4y+3x^3-y^2+xy-x^2 \\ 5x^3-2x+y-15+y^2 \\ \hline 3x^2-xy-14+2y+12x^3-2x \end{array}$$

Here $4x^2-x^2=3x^2$

$-2xy+xy=-xy$

$+1-15=-14$

$-3y+4y+y=+2y$

$+4x^3+3x^3+5x^3=+12x^3$

$-y^2+y^2=0$

$-2x=-2x.$

IV.

SUBTRACTION.

18. If it were required to subtract $5-2$ (i.e. 3) from 9, it is evident that the remainder would be *greater* by 2, than if 5 only were subtracted. For the same reason, if $b-c$ were

were subtracted from a, the remainder would be greater by c, than if b only were subtracted. Now, if b is subtracted from a, the remainder is $a-b$; and consequently, if $b-c$ be subtracted from a, the remainder will be $a-b+c$. Hence this general Rule for the subtraction of algebraic quantities; "Change the signs of the quantities *to be subtracted*, and "then place them one after another, as in Addition."

Ex. 1.

From $5a+3x-2b$, take $2c-4y$. The quantity to be subtracted *with its signs changed*, is $-2c+4y$; therefore the remainder is $5a+3x-2b-2c+4y$.

Ex. 2.

From $7x^2-2x+5$, take $3x^2+5x-1$.

The remainder is $7x^2-2x+5\ \ -3x^2-5x+1$,

or $7x^2-3x^2-2x-5x+5\ \ +1=4x^2-7x+6$.

But when *like* quantities are to be subtracted from each other, as in Ex. 2., the better way is to set one row under the other, and apply the following Rule; "*Conceive* the "signs of the *quantities to be subtracted* to be *changed*, and "then proceed as in Addition."

	Ex. 3.	Ex. 4.	Ex. 5.
From	$7x^2-2x+5$	$12a^2-3a+\ b-1$	$5y^2-4y+3a$
Subtract	$3x^2+5x-1$	$6a^2+\ a-2b+3$	$6y^2-4y-\ a$
Remainder	$4x^2-7x+6$	$6a^2-4a+3b-4$	$-y^2\ \ *\ \ +4a$

	Ex. 6.	Ex. 7.	Ex. 8.
From	$7xy+2x-3y$	$14x+y-z-\ 5$	$13x^3-2x^2+7$
Subtract	$2xy-\ x+\ y$	$x+y+z-11$	$-x^3+\ x^2-6$
Remainder			

MULTI-

V.

MULTIPLICATION.

19. In the multiplication of algebraic quantities, the four following Rules must be observed.

I. When quantities having *like* signs are multiplied together, the sign of the *product* will be +; and if their signs are *unlike*, the sign of the *product* will be −.*

II. The coefficients of the *factors* must be multiplied together, to form the coefficient of the *product*.

III. The

---

* This rule for the multiplication of the Signs may be thus explained:

I. If $+a$ is to be multiplied by $+b$, it means, that $+a$ is to be *added* to itself as often as there are units in b, and consequently the product will be $+ab$.

II. If $-a$ is to be multiplied by $+b$, it means, that $-a$ is to be *added* to itself as often as there are units in b, and therefore the product is $-ab$.

III. If $+a$ is to be multiplied by $-b$, it means, that $+a$ is to be *subtracted* as often as there are units in b, and consequently the product is $-ab$.

IV. If $-a$ is to be multiplied by $-b$, it means, that $-a$ is to be *subtracted* as often as there are units in b; and, since to *subtract a negative quantity* is the same as *to add a positive one*, the product will be $+ab$.

Or, these Four Rules might be all comprehended in *one*; thus,

To multiply $a-b$ by $c-d$, is to add $a-b$ to itself as often as there are units in $c-d$; now this is done by *adding* it *c times*, and *subtracting* it *d times*;

But $a-b$, *added c times* . . $= ac-bc$,

and $a-b$, *subtracted d times* $= -ad+bd$,

$\therefore \overline{a-b} \times \overline{c-d}$ . . . . . . $= ac-bc-ad+bd$.

i. e. $+a \times +c = +ac$

$-b \times +c = -bc$

$+a \times -d = -ad$

$-b \times -d = +bd$.