

**INTRODUCTORY COURSE IN  
DIFFERENTIAL EQUATIONS  
FOR STUDENTS IN CLASSICAL  
AND ENGINEERING COLLEGES**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649615582

Introductory Course in Differential Equations for Students in Classical and Engineering Colleges  
by D. A. Murray

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.  
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

[www.triestepublishing.com](http://www.triestepublishing.com)

**D. A. MURRAY**

**INTRODUCTORY COURSE IN  
DIFFERENTIAL EQUATIONS  
FOR STUDENTS IN CLASSICAL  
AND ENGINEERING COLLEGES**



*Alex. de Zee*  
INTRODUCTORY COURSE

IN

## DIFFERENTIAL EQUATIONS

*FOR STUDENTS IN CLASSICAL AND  
ENGINEERING COLLEGES*

BY

D. A. MURRAY, B.A., Ph.D.

FORMERLY SCHOLAR AND FELLOW OF JOHNS HOPKINS UNIVERSITY  
INSTRUCTOR IN MATHEMATICS IN CORNELL UNIVERSITY

LONGMANS, GREEN, AND CO.

LONDON AND BOMBAY

1897

COPYRIGHT, 1907,  
BY LONGMANS, GREEN, AND CO.

---

ALL RIGHTS RESERVED.

---

Typography by J. S. Cushing & Co., Norwood, Mass., U. S. A.

## PREFACE.

---

THE aim of this work is to give a brief exposition of some of the devices employed in solving differential equations. The book presupposes only a knowledge of the fundamental formulae of integration, and may be described as a chapter supplementary to the elementary works on the integral calculus.

The needs of two classes of students, with whom the author has been brought into contact in the course of his experience as a teacher, have determined the character of the work. For the sake of students of physics and engineering who wish to use the subject as a tool, and have little time to devote to general theory, the theoretical explanations have been made as brief as is consistent with clearness and sound reasoning, and examples have been worked in full detail in almost every case. Practical applications have also been constantly kept in mind, and two special chapters dealing with geometrical and physical problems have been introduced.

The other class for which the book is intended is that of students in the general courses in Arts and Science, who have more time to gratify any interest they may feel in this subject, and some of whom may be intending to proceed to the study of the higher mathematics. For these students, notes have

been inserted in the latter part of the book. Some of the notes contain the demonstrations of theorems which are referred to, or partially proved, in the first part of the work. If these discussions were given in full in the latter place, they would probably tend to discourage a beginner. Accordingly, it has been thought better to delay the rigorous proof of several theorems until the student has acquired some degree of familiarity with the working of examples.

Throughout the book are many historical and biographical notes, which it is hoped will prove interesting. In order that beginners may have a larger and better conception of the subject, it seemed right to point out to them some of the most important lines of development of the study of differential equations, and notes have been given which have this object in view. For this purpose, also, a few articles have been placed in the body of the text. These articles refer to Riccati's, Bessel's, Legendre's, Laplace's, and Poisson's equations, and the equation of the hypergeometric series, which are forms that properly lie beyond the scope of an introductory work.

In many cases in which points are dismissed in the brief manner necessary in a work of this kind, references are given where fuller explanations and further developments may be found. These references are made, whenever possible, to sources easily accessible to an ordinary student, and especially to the standard treatises, in English, of Boule, Forsyth, and Johnson.

For students who can afford but a minimum of time for this study, the essential articles of a short course are indicated after the table of contents.



---

Of the examples not a few are original, and many are taken from examination papers of leading universities. There is also a large number of examples, which, either by reason of their frequent use in mechanical problems or their excellence as examples *per se*, are common to all elementary text-books on differential equations.

There remains the pleasant duty of making confession of my indebtedness.

In preparing this book, I have consulted many works and memoirs; and, in particular, have derived especial help for the principal part of the work from the treatises of Boole, Forsyth, and Johnson, and from the chapters on Differential Equations in the works of De Morgan, Moigno, Houel, Laurent, Boussinesq, and Mansion. I have in addition to acknowledge suggestions received from Byerly's "Key to the Solution of Differential Equations" published in his *Integral Calculus*, Osborne's *Examples and Rules*, and from the treatises of Williamson, Edwards, and Stegemann on the Calculus. Use has also been made of notes of a course of lectures delivered by Professor David Hilbert at Göttingen. Suggestions and material for many of the historical and other notes have also been received from the works of Craig, Jordan, Picard, Goursat, Koenigsberger, and Schlesinger on Differential Equations; from Byerly's *Fourier's Series and Spherical Harmonics*, Cajori's *History of Mathematics*, and from the chapters on Hyperbolic Functions, Harmonic Functions, and the History of Modern Mathematics in Merriman and Woodward's *Higher Mathematics*. The mechanical and physical examples have been obtained from Tait and Steele's *Dynamics of a Particle*, Ziwet's *Mechanics*, Thomson and Tait's *Natural Philosophy*,

Emtage's *Mathematical Theory of Electricity and Magnetism*,  
Bedell and Crehore's *Alternating Currents*, and Bedell's *Prin-  
ciples of the Transformer*. These and many other acknowledg-  
ments will be found in various parts of the book.

To the friends who have encouraged and aided me in this  
undertaking, I take this opportunity of expressing my grati-  
tude. And first and especially to Professor James McMahon  
of Cornell University, whose opinions, advice, and criticisms,  
kindly and freely given, have been of the greatest service to  
me. I have also to thank Professors E. Merritt and F. Bedell  
of the department of physics, and Professor Tanner, Mr.  
Saurel, and Mr. Allen of the department of mathematics at  
Cornell for valuable aid and suggestions. Professor McMahon  
and Mr. Allen have also assisted me in revising the proof-sheets  
while the work was going through the press. To Miss H. S.  
Poole and Mr. M. Macneil, graduate students at Cornell, I am  
indebted for the verification of many of the examples.

D. A. MURRAY.

CORNELL UNIVERSITY,  
April, 1897.

# CONTENTS.

## EQUATIONS INVOLVING TWO VARIABLES.

### CHAPTER I.

#### DEFINITIONS. FORMATION OF A DIFFERENTIAL EQUATION.

ART.	PAGE
1. Ordinary and partial differential equations. Order and degree . . . . .	1
2. Solutions and constants of integration . . . . .	2
3. The derivation of a differential equation . . . . .	4
4. Solutions, general, particular, singular . . . . .	6
5. Geometrical meaning of a differential equation of the first order and degree . . . . .	8
6. Geometrical meaning of a differential equation of a degree or an order higher than the first . . . . .	9
Examples on Chapter I. . . . .	11

### CHAPTER II.

#### EQUATIONS OF THE FIRST ORDER AND OF THE FIRST DEGREE.

8. Equations of the form $f_1(x)dx + f_2(y)dy = 0$ . . . . .	14
9. Equations homogeneous in $x$ and $y$ . . . . .	15
10. Non-homogeneous equations of the first degree in $x$ and $y$ . . . . .	16
11. Exact differential equations . . . . .	17
12. Condition that an equation of the first order be exact . . . . .	18
13. Rule for finding the solution of an exact differential equation . . . . .	19
14. Integrating factors . . . . .	21
15. The number of integrating factors is infinite . . . . .	21
16. Integrating factors found by inspection . . . . .	22
17. Rules for finding integrating factors. Rules I. and II. . . . .	23
18. Rules III. and IV. . . . .	24