

**PLANE
TRIGONOMETRY
AND APPLICATIONS**

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Plane Trigonometry and Applications by E. J. Wilczynski & H. E. Slaughter

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E. J. WILCZYNSKI & H. E. SLAUGHT

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APPLICATIONS

BY

revised edition
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THE UNIVERSITY OF CHICAGO

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PREFACE

THE characteristic features of this book may be summarized as follows:

1. *The method of presentation is thoroughly heuristic.* This enables the student to get a firm grasp of the subject by teaching him to recognize the fundamental ideas which underlie and unify the separate steps of the mathematical argument, instead of confusing him by a disconnected dogmatic statement of isolated facts.

2. *The book is divided into two parts.* The first part is devoted to the theoretical and numerical solution of triangles. The second part treats of the functions of the general angle, their addition theorems and other properties, together with applications to simple harmonic curves, simple harmonic and wave motion, and harmonic analysis. Part One is also published separately and is well adapted for use in secondary schools. The complete book is intended for the freshman course in colleges.

3. *The discussion of the solution of triangles, in Part One, is not interrupted by any digressions about coördinate systems, addition theorems, and the like.* It has been thought desirable to postpone to the second part the consideration of all of these matters, which are indeed important but unnecessary for the solution of triangles.

4. *The definitions for the functions of an obtuse angle* have been made to grow organically out of the needs of the problem of solving triangles, in a way which seems both simple and natural, and which at the same time illustrates an important principle of mathematical procedure.

5. *The whole theory of triangles has been unified* by giving a central position to the area problem. As a consequence, almost all of the necessary equations present themselves spontaneously and in a connected fashion. The law of tangents is the only one which causes any difficulty in this respect. But the law of tangents also has been made to submit to a heuristic treatment, by

introducing the notion of the *form-ratio* of a triangle, and combining this notion with a direct geometric proof of the formulæ for $\sin A + \sin B$ and $\sin A - \sin B$.

6. *The numerical aspect of the work has been discussed very fully.* Directions for computation are given in great detail; most of the common sources of error are pointed out; and methods for detecting and correcting them are indicated. After a thorough discussion of the significance of the number of decimal places needed in a computation, the student is urged to train and use his judgment on this matter. He is given an opportunity to do this by supplying him with complete five- and three-place tables and a partial set of four-place tables.

7. *The slide rule is explained* with considerable detail and its use recommended. A number of other labor-saving devices are discussed.

8. *The examples have been selected with great care.* Examples without real significance have been avoided, and the numbers have been chosen so as not to lead to five-place calculations when such a show of accuracy would be absurd. Special efforts have been made to word the examples in such a way as to avoid ambiguity.

9. *The applications cover a wider field than usual,* and include problems in heights and distances, surveying, navigation, engineering, astronomy, and physics. But the examples involving such applications are not, as in most texts, introduced at random and without previous explanation. Every notion which is required for the solution of any example in the book is fully explained on the spot or in some earlier portion of the text.

10. *The use of a few new terms,* such as the *standard position* of an angle, odd and even *cardinal angles*, has helped to simplify materially the statement of a number of important results.

11. *The addition formulæ are presented in two different ways.* The first, more elementary method, is made to yield the general result by the help of mathematical induction. The second method, based on the notions of directed lines, line-segments, and angles, appears here in a very simple and elegant form.

12. *The articles on harmonic and wave motion* tend to show the student that Trigonometry has other applications besides the solution of triangles.

13. *A considerable amount of historical matter has been introduced, not in the form of detached historical notes, but organically connected with the topic under discussion. Most of this matter was gathered from BRAUNMÜHL's *Vorlesungen über die Geschichte der Trigonometrie*. Professors CAJORI and KARPINSKI have kindly answered some questions of a historical nature about which we were in doubt.*

14. *The type and the manner of spacing used in the tables are the results of a number of experiments, the object being to produce a set of tables which should be as pleasant to the eye as possible. The tables are bound separately for various reasons. In order to make them easily legible, a certain size of page was necessary, and it was thought undesirable to use so large a page for the text itself. In the second place, it is a great advantage for the student if he can have his text and his tables open before him at the same time. In the third place, it is often desirable, in examinations, to allow students to use their tables without their books. Finally, a separation of the tables and text makes it easy to use this text with other tables, or these tables with other texts, thus providing a maximum of elasticity in organizing a course.*

Many of the older texts on Trigonometry have been consulted during the preparation of this book, and the attempt has been made to learn from all of them. The works of SERRET, LÜBSEN, WIEGAND, CROCKETT, MORITZ, HALL and FRINK have been especially helpful. A few purely numerical examples have actually been taken from these and other texts without change, so as to reduce somewhat the task of computing the answers, and at the same time to make the answers more trustworthy. Most of the examples, however, are new; many of them are new in kind.

In conclusion, the author and editor wish to acknowledge their indebtedness to their colleagues at the University of Chicago for various helpful suggestions and criticisms.

E. J. WILCZYNSKI.
H. E. SLAUGHT, EDITOR.



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