

**SOLID GEOMETRY  
WITH PROBLEMS  
AND APPLICATIONS**

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Solid Geometry with Problems and Applications by H. E. Slaught & N. J. Lennes

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**H. E. SLAUGHT & N. J. LENNES**

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**Bonaventura Cavalieri** (1598-1647) was one of the most influential mathematicians of his time. He was chiefly noted for his invention of the so-called "Principle of Indivisibles" by which he derived areas and volumes. See pages 134 and 202.

frontis

SOLID GEOMETRY  
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PROBLEMS AND APPLICATIONS

*REVISED EDITION*

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## PREFACE

In re-writing the Solid Geometry the authors have consistently carried out the distinctive features described in the preface of the Plane Geometry. Mention is here made only of certain matters which are particularly emphasized in the Solid Geometry.

Owing to the greater maturity of the pupils it has been possible to make the logical structure of the Solid Geometry more prominent than in the Plane Geometry. The axioms are stated and applied at the precise points where they are to be used. Theorems are no longer quoted in the proofs but are only referred to by paragraph numbers; while with increasing frequency the student is left to his own devices in supplying the reasons and even in filling in the logical steps of the argument. For convenience of reference the axioms and theorems of plane geometry which are used in the Solid Geometry are collected in the Introduction.

In order to put the essential principles of solid geometry, together with a reasonable number of applications, within limited bounds (156 pages), certain topics have been placed in an Appendix. This was done in order to provide a minimum course in convenient form for class use and not because these topics, Similarity of Solids and Applications of Projection, are regarded as of minor importance. In fact, some of the examples under these topics are among the most interesting and concrete in the text. For example, see pages 170-172, 177, 183-184.

The exercises in the main body of the text are carefully graded as to difficulty and are not too numerous to be easily performed. The concepts of three-dimensional space are made



clear and vivid by many simple illustrations and questions under the suggestive headings "Sight Work." This plan of giving many and varied simple exercises, so effective in the Plane Geometry, is still more valuable in the Solid Geometry where the visualizing of space relations is difficult for many pupils.

The treatment of incommensurables throughout the body of this text, both Plane and Solid, is believed to be sane and sensible. In each case, a frank assumption is made as to the existence of the concept in question (length of a curve, area of a surface, volume of a solid) and of its realization for all practical purposes by the approximation process. Then, for theoretical completeness, rigorous proofs of these theorems are given in Appendix III, where the theory of limits is presented in far simpler terminology than is found in current text-books and in such a way as to leave nothing to be unlearned or compromised in later mathematical work.

Acknowledgment is due to Professor David Eugene Smith for the use of portraits from his collection of portraits of famous mathematicians.

H. E. SLAUGHT  
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CHICAGO AND MISSOULA,  
May, 1919.

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