THE GEOMETRY OF CYCLOIDS. A TREATISE ON THE CYCLOID AND ALL FORMS OF CYCLOIDAL CURVES, AND ON THE USE OF SUCH CURVES IN DEALING WITH THE MOTIONS OF PLANETS, COMETS, &C., AND OF MATTER PROJECTED FROM THE SUN

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The Geometry of Cycloids. A Treatise on the Cycloid and All Forms of Cycloidal Curves, and on the Use of Such Curves in Dealing with the Motions of Planets, Comets, &c., and of Matter Projected from the Sun by Richard A. Proctor

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RICHARD A. PROCTOR

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THE

GEOMETRY OF CYCLOIDS

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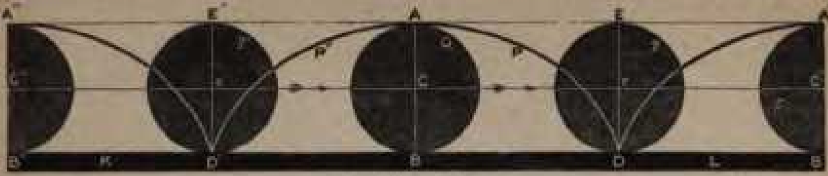
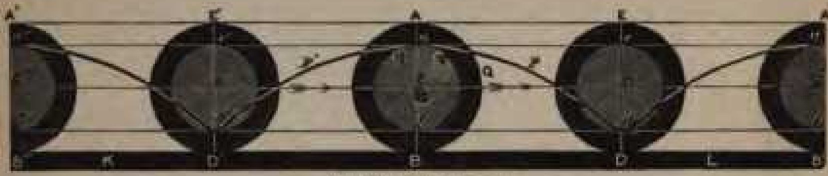
FIG. 1.



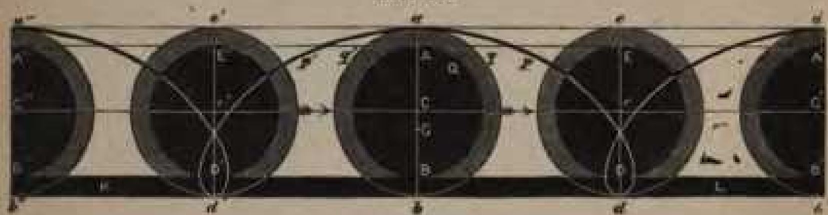
THE RIGHT CYCLOID.

FIG. 45.



THE PROLATE CYCLOID.

FIG. 46.

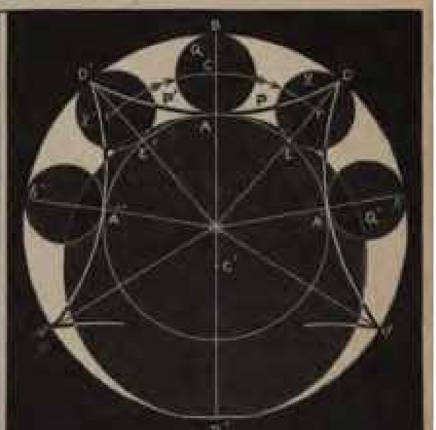


THE CURTATE CYCLOID.

FIG. 19.

THE EPICYCLOID.

FIG. 20.



THE HYPOCYCLOID.

A TREATISE ON

THE CYCLOID

AND ALL FORMS OF

CYCLOIDAL CURVES

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AND OF

MATTER PROJECTED FROM THE SUN

BY

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PREFACE.

THIS WORK deals primarily with the geometry of *cycloids*, curves traced out by a point in a circle rolling on a straight line, or on or within another circle, and *trochoids* (or hoop-curves), curves traced out by a point within or without a circle so rolling.

Although the invention of the cycloid is attributed to Galileo, it is certain that the family of curves to which the cycloid belongs had been known, and some of the properties of such curves investigated, nearly two thousand years before Galileo's time, if not earlier. For ancient astronomers explained the motion of the planets by supposing that each planet travels uniformly round a circle whose centre travels uniformly round another circle. By suitably selecting radii for such circles, and velocities for the uniform motions in them, every form of epicyclic curve can be obtained, including the epicycloid and the hypocycloid. When the radius of the fixed circle is indefinitely enlarged, or, in other words, when the centre of the moving circle advances uniformly in a straight line, the curve traced out by the moving point becomes a *trochoid*, and may either be a *prolate*, a *right*, or a *curtate cycloid*, according as the velocity of the moving centre is greater, equal, or less than the velocity of the point around that centre. Lastly, if the radius of the moving circle is indefinitely enlarged, so that a straight line is carried uniformly round a centre while a point travels uniformly along the line, the curve traced out becomes a spiral of the family to which belong the spiral of Archimedes and the involute of the circle.

It is of these curves, which are all included under the general name epicyclical curves, that I treat in the present volume, though the cycloid, epicycloid, hypocycloid, and trochoid are more fully dealt with, in their geometrical aspect, than the epitrochoidal and spiral members of the epicyclic family.

Ancient geometers were not very successful in their attempts to investigate any of these curves. It is strange indeed to find a mathematician even of Galileo's force so far foiled by the common cycloid as to be reduced to the necessity of weighing paper figures of the curve in order to determine its area. Pascal dealt more successfully with this and other problems. Yet he seems to have regarded their relations as of sufficient difficulty to be selected for his famous challenge to mathematicians, to try whether a priest who had long given up the study of mathematics was not a match for mathematicians at their own weapons. The argument, in so far as it was intended to prove the soundness of Pascal's faith, was feeble enough. But the failure, or partial failure, of many who attacked his problems, is noteworthy. We find, for instance, that Roberval laboured for six years over the quadrature of the cycloid, and only succeeded at last in solving it by the comparatively clumsy method indicated at p. 199, inventing a new curve for the purpose. It will be seen that in the present work this famous problem comes very early (Prop. III., pp. 5, 6), and is made to depend on the fundamental (and obvious) relation of the cycloidal ordinates. The method—which so far as I know is a new one—is extended to the epicycloid, hypocycloid, trochoid, epitrochoid, and hypotrochoid. It will be found that, in all, thirteen distinct methods of solving the problem geometrically are either given in full or indicated (seven of these methods being new so far as I know), while seven independent methods are indicated for determining the area of the epicycloid and hypocycloid (of which five are new), besides one method (see footnote, p. 50) derived from the properties of the cycloid. After the first demonstration of the