A SHORT TABLE OF INTEGRALS. SECOND REVISED EDITION

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Trieste

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SHORT TABLE OF INTEGRALS

BY

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Since I cannot hope that these formulas are wholly free from misprints, I shall be grateful to any person who will call my attention to such errors as he may discover.

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TABLE OF INTEGRALS.

I. FUNDAMENTAL FORMS.

1. $\int a\,dx = ax.$

2. $\int af(x)\,dx = a\int f(x)\,dx.$

3. $\int \frac{dx}{x} = \log x.$

4. $\int x^m dx = \frac{x^{m+1}}{m+1}$, when m is different from -1.

5. $\int e^x dx = e^x.$

6. $\int a^x \log a\,dx = a^x.$

7. $\int \frac{dx}{1+x^2} = \tan^{-1}x$, or $-\operatorname{ctn}^{-1}x.$

8. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x$, or $-\cos^{-1}x.$

9. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x$, or $-\csc^{-1}x.$

10. $\int \frac{dx}{\sqrt{2x-x^2}} = \operatorname{versin}^{-1}x$, or $-\operatorname{coversin}^{-1}x.$

11. $\int \cos x\, dx = \sin x$, or $-$ coversin x.

12. $\int \sin x\, dx = -\cos x$, or versin x.

13. $\int \operatorname{ctn} x\, dx = \log \sin x$.

14. $\int \tan x\, dx = -\log \cos x$.

15. $\int \tan x \sec x\, dx = \sec x$.

16. $\int \sec^2 x\, dx = \tan x$.

17. $\int \csc^2 x\, dx = -\operatorname{ctn} x$.

In the following formulas, u, v, w, and y represent any functions of x:

18. $\int (u + v + w + \text{etc.})\, dx = \int u\, dx + \int v\, dx + \int w\, dx + \text{etc.}$

19 *a*. $\int u\, dv = uv - \int v\, du$.

19 *b*. $\int u \frac{dv}{dx}\, dx = uv - \int v \frac{du}{dx}\, dx$.

20. $\int f(y)\, dx = \int \frac{f(y)\, dy}{\frac{dy}{dx}}$.

II. RATIONAL ALGEBRAIC FUNCTIONS.

A. — Expressions Involving $(a + bx)$.

The substitution of y or z for x, where $y \equiv a + bx$, $z \equiv (a + bx) / x$, gives

21. $\int (a + bx)^m dx = \frac{1}{b} \int y^m dy.$

22. $\int x (a + bx)^m dx = \frac{1}{b^2} \int y^m (y - a)\, dy.$

23. $\int x^n (a + bx)^m dx = \frac{1}{b^{n+1}} \int y^m (y - a)^n dy.$

24. $\int \frac{x^n dx}{(a + bx)^m} = \frac{1}{b^{n+1}} \int \frac{(y - a)^n dy}{y^m}.$

25. $\int \frac{dx}{x^n (a + bx)^m} = - \frac{1}{a^{m+n-1}} \int \frac{(z - b)^{m+n-2} dz}{z^m}.$

Whence

26. $\int \frac{dx}{a + bx} = \frac{1}{b} \log (a + bx).$

27. $\int \frac{dx}{(a + bx)^2} = - \frac{1}{b (a + bx)}.$

28. $\int \frac{dx}{(a + bx)^3} = - \frac{1}{2 b (a + bx)^2}.$

29. $\int \frac{x\, dx}{a + bx} = \frac{1}{b^2} [a + bx - a \log (a + bx)].$

30. $\int \frac{x\, dx}{(a + bx)^2} = \frac{1}{b^2} \left[\log (a + bx) + \frac{a}{a + bx} \right].$

31. $\int \frac{x\,dx}{(a+bx)^3} = \frac{1}{b^2}\left[-\frac{1}{a+bx} + \frac{a}{2(a+bx)^2}\right].$

32. $\int \frac{x^3\,dx}{a+bx} = \frac{1}{b^3}\left[\tfrac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2\log(a+bx)\right].$

33. $\int \frac{x^2\,dx}{(a+bx)^2} = \frac{1}{b^3}\left[a+bx-2a\log(a+bx)-\frac{a^2}{a+bx}\right].$

34. $\int \frac{dx}{x(a+bx)} = -\frac{1}{a}\log\frac{a+bx}{x}.$

35. $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2}\log\frac{a+bx}{x}.$

36. $\int \frac{(a+bx)\,dx}{a'+b'x} = \frac{bx}{b'} + \frac{ab'-a'b}{b'^2}\log(a'+b'x).$

37. $$\int (a+bx)^n(a'+b'x)^m\,dx = \frac{1}{(m+n+1)b}\Big((a+bx)^{n+1}(a'+b'x)^m$$
$$-m(ab'-a'b)\int (a+bx)^n(a'+b'x)^{m-1}dx\Big).$$

38. $$\int \frac{(a+bx)^n\,dx}{(a'+b'x)^m} = -\frac{1}{(m-1)(ab'-a'b)}\left(\frac{(a+bx)^{n+1}}{(a'+b'x)^{m-1}}\right.$$
$$\left.+(m-n-2)b\int\frac{(a+bx)^n\,dx}{(a'+b'x)^{m-1}}\right)$$
$$= -\frac{1}{(m-n-1)b'}\left(\frac{(a+bx)^n}{(a'+b'x)^{m-1}}\right.$$
$$\left.+n(ab'-a'b)\int\frac{(a+bx)^{n-1}\,dx}{(a'+b'x)^m}\right)$$
$$= -\frac{1}{(m-1)b'}\left(\frac{(a+bx)^n}{(a'+b'x)^{m-1}} - nb\int\frac{(a+bx)^{n-1}\,dx}{(a'+b'x)^{m-1}}\right).$$

39. $\displaystyle\int \frac{dx}{(a+bx)(a'+b'x)} = \frac{1}{ab'-a'b}\cdot\log\frac{a'+b'x}{a+bx}.$

40. $\displaystyle\int \frac{dx}{(a+bx)^n(a'+b'x)^m}$

$$= \frac{1}{(m-1)(ab'-a'b)}\left(\frac{1}{(a+bx)^{n-1}(a'+b'x)^{m-1}} - (m+n-2)\,b\int\frac{dx}{(a+bx)^n(a'+b'x)^{m-1}}\right).$$

41. $\displaystyle\int \frac{x\,dx}{(a+bx)(a'+b'x)}$

$$= \frac{1}{ab'-a'b}\left(\frac{a}{b}\log(a+bx) - \frac{a'}{b'}\log(a'+b'x)\right).$$

42. $\displaystyle\int \frac{dx}{(a+bx)^2(a'+b'x)}$

$$= \frac{1}{ab'-a'b}\left(\frac{1}{a+bx} + \frac{b'}{ab'-a'b}\log\frac{a'+b'x}{a+bx}\right).$$

43. $\displaystyle\int \frac{x\,dx}{(a+bx)^2(a'+b'x)}$

$$= \frac{-a}{b(ab'-a'b)(a+bx)} - \frac{a'}{(ab'-a'b)^2}\log\frac{a'+b'x}{a+bx}.$$

44. $\displaystyle\int \frac{x^2\,dx}{(a+bx)^2(a'+b'x)} = \frac{a^2}{b^2(ab'-a'b)(a+bx)}$

$$+\frac{1}{(ab'-a'b)^2}\left[\frac{a'^2}{b'}\log(a'+b'x) + \frac{a(ab'-2a'b)}{b^2}\log(a+bx)\right].$$

45. $\displaystyle\int (a+bx)^{\frac{1}{n}}\,dx = \frac{n}{(n+1)b}(a+bx)^{\frac{n+1}{n}}.$

46. $\displaystyle\int \frac{dx}{(a+bx)^{\frac{1}{n}}} = \frac{n}{(n-1)b}(a+bx)^{\frac{n-1}{n}}.$