

**A SHORT TABLE OF
INTEGRALS. SECOND
REVISED EDITION**

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A Short Table of Integrals. Second Revised Edition by B. O. Peirce

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A

SHORT TABLE OF INTEGRALS

BY

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SECOND REVISED EDITION

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*Since I cannot hope that these formulas are wholly free
from misprints, I shall be grateful to any person who will
call my attention to such errors as he may discover.*

B. O. PEIRCE,
Harvard University, Cambridge.

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TABLE OF INTEGRALS.

I FUNDAMENTAL FORMS

1. $\int a dx = ax.$
2. $\int af(x) dx = a \int f(x) dx.$
3. $\int \frac{dx}{x} = \log x.$
4. $\int x^m dx = \frac{x^{m+1}}{m+1},$ when m is different from $-1.$
5. $\int e^x dx = e^x.$
6. $\int a^x \log a dx = a^x.$
7. $\int \frac{dx}{1+x^2} = \tan^{-1}x,$ or $-\cot^{-1}x.$
8. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x,$ or $-\cos^{-1}x.$
9. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x,$ or $-\csc^{-1}x.$
10. $\int \frac{dx}{\sqrt{2x-x^2}} = \text{versin}^{-1}x,$ or $-\text{coversin}^{-1}x.$

$$11. \int \cos x \, dx = \sin x, \text{ or } -\text{coversin } x.$$

$$12. \int \sin x \, dx = -\cos x, \text{ or versin } x.$$

$$13. \int \text{ctn } x \, dx = \log \sin x.$$

$$14. \int \tan x \, dx = -\log \cos x.$$

$$15. \int \tan x \sec x \, dx = \sec x.$$

$$16. \int \sec^2 x \, dx = \tan x.$$

$$17. \int \csc^2 x \, dx = -\text{ctn } x.$$

In the following formulas, u , v , w , and y represent any functions of x :

$$18. \int (u + v + w + \text{etc.}) \, dx = \int u \, dx + \int v \, dx + \int w \, dx + \text{etc.}$$

$$19 a. \int u \, dv = uv - \int v \, du.$$

Vit

$$19 b. \int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx.$$

$$20. \int f(y) \, dx = \int \frac{f(y) \, dy}{\frac{dy}{dx}}.$$

II. RATIONAL ALGEBRAIC FUNCTIONS.

A. — EXPRESSIONS INVOLVING $(a + bx)$.

The substitution of y or z for x , where $y \equiv a + bx$,
 $z \equiv (a + bx)/x$, gives

$$21. \int (a + bx)^n dx = \frac{1}{b} \int y^n dy.$$

$$22. \int x(a + bx)^n dx = \frac{1}{b^2} \int y^n (y - a) dy.$$

$$23. \int x^n (a + bx)^n dx = \frac{1}{b^{n+1}} \int y^n (y - a)^n dy.$$

$$24. \int \frac{x^n dx}{(a + bx)^n} = \frac{1}{b^{n+1}} \int \frac{(y - a)^n dy}{y^n}.$$

$$25. \int \frac{dx}{x^n (a + bx)^n} = -\frac{1}{a^{m+n-1}} \int \frac{(z - b)^{m+n-2} dz}{z^m},$$

Whence

$$26. \int \frac{dx}{a + bx} = \frac{1}{b} \log(a + bx).$$

$$27. \int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}.$$

$$28. \int \frac{dx}{(a + bx)^3} = -\frac{1}{2b(a + bx)^2}.$$

$$29. \int \frac{x dx}{a + bx} = \frac{1}{b^2} [a + bx - a \log(a + bx)].$$

$$30. \int \frac{x dx}{(a + bx)^2} = \frac{1}{b^2} \left[\log(a + bx) + \frac{a}{a + bx} \right].$$

- $$31. \int \frac{x dx}{(a+bx)^2} = \frac{1}{b^2} \left[-\frac{1}{a+bx} + \frac{a}{2(a+bx)^2} \right].$$
- $$32. \int \frac{x^2 dx}{a+bx} = \frac{1}{b^2} \left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2 \log(a+bx) \right].$$
- $$33. \int \frac{x^2 dx}{(a+bx)^2} = \frac{1}{b^2} \left[a+bx - 2a \log(a+bx) - \frac{a^2}{a+bx} \right].$$
- $$34. \int \frac{dx}{x(a+bx)} = -\frac{1}{a} \log \frac{a+bx}{x}.$$
- $$35. \int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2} \log \frac{a+bx}{x}.$$
- $$36. \int \frac{(a+bx) dx}{a'+b'x} = \frac{bx}{b'} + \frac{ab' - a'b}{b'^2} \log(a'+b'x).$$
- $$37. \int (a+bx)^n (a'+b'x)^m dx = \frac{1}{(m+n+1)b} \left((a+bx)^{n+1} (a'+b'x)^m \right. \\ \left. - m(ab' - a'b) \int (a+bx)^n (a'+b'x)^{m-1} dx \right).$$
- $$38. \int \frac{(a+bx)^n dx}{(a'+b'x)^m} = -\frac{1}{(m-1)(ab' - a'b)} \left(\frac{(a+bx)^{n+1}}{(a'+b'x)^{m-1}} \right. \\ \left. + (m-n-2)b \int \frac{(a+bx)^n dx}{(a'+b'x)^{m-1}} \right) \\ = -\frac{1}{(m-n-1)b'} \left(\frac{(a+bx)^n}{(a'+b'x)^{m-1}} \right. \\ \left. + n(ab' - a'b) \int \frac{(a+bx)^{n-1} dx}{(a'+b'x)^m} \right) \\ = -\frac{1}{(m-1)b'} \left(\frac{(a+bx)^n}{(a'+b'x)^{m-1}} - nb \int \frac{(a+bx)^{n-1} dx}{(a'+b'x)^{m-1}} \right).$$

$$39. \int \frac{dx}{(a+bx)(a'+b'x)} = \frac{1}{ab'-a'b} \cdot \log \frac{a'+b'x}{a+bx}.$$

$$40. \int \frac{dx}{(a+bx)^n (a'+b'x)^m} \\ = \frac{1}{(m-1)(ab'-a'b)} \left(\frac{1}{(a+bx)^{n-1} (a'+b'x)^{m-1}} \right. \\ \left. - (m+n-2)b \int \frac{dx}{(a+bx)^n (a'+b'x)^{m-1}} \right).$$

$$41. \int \frac{x dx}{(a+bx)(a'+b'x)} \\ = \frac{1}{ab'-a'b} \left(\frac{a}{b} \log(a+bx) - \frac{a'}{b'} \log(a'+b'x) \right).$$

$$42. \int \frac{dx}{(a+bx)^2 (a'+b'x)} \\ = \frac{1}{ab'-a'b} \left(\frac{1}{a+bx} + \frac{b'}{ab'-a'b} \log \frac{a'+b'x}{a+bx} \right).$$

$$43. \int \frac{x dx}{(a+bx)^2 (a'+b'x)} \\ = \frac{-a}{b(ab'-a'b)(a+bx)} - \frac{a'}{(ab'-a'b)^2} \log \frac{a'+b'x}{a+bx}.$$

$$44. \int \frac{x^2 dx}{(a+bx)^2 (a'+b'x)} = \frac{a^2}{b^2(ab'-a'b)(a+bx)} \\ + \frac{1}{(ab'-a'b)^2} \left[\frac{a^2}{b'} \log(a'+b'x) + \frac{a(ab'-2a'b)}{b^2} \log(a+bx) \right].$$

$$45. \int (a+bx)^{\frac{1}{n}} dx = \frac{n}{(n+1)b} (a+bx)^{\frac{n+1}{n}}.$$

$$46. \int \frac{dx}{(a+bx)^{\frac{1}{n}}} = \frac{n}{(n-1)b} (a+bx)^{\frac{n-1}{n}}.$$