APPLICATIONS OF PLANE AND SPHERICAL TRIGONOMETRY. PP.208-295

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EUGENE L. RICHARDS

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APPLICATIONS

OF

PLANE AND SPHERICAL

TRIGONOMETRY.

BY

EUGENE L. RICHARDS, B. A.,

ASSISTANT PROFESSOR OF MATHEMATICS IN YALE COLLEGE.

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PREFACE.

In this book nothing more has been attempted than an introduction to the subjects specified, and a logical application of the principles of Trigonometry to them; so that a student, faithfully mastering these chapters, may understandingly pursue the subjects in larger treatises.

In the preparation of the text, and in the choice of examples in the chapter on Surveying, the author acknowledges special obligation to "Gillespie's Land Surveying," from which he has received valuable hints. "Bowditch's Navigator" has been frequently consulted, both for the definitions and for the materials of the text in the chapter on Navigation. In both these chapters, however, the author has used his own methods in presenting the subjects.

Articles referred to by number are to be found in the author's "Elements of Plane and Spherical Trigonometry," already published. Other references are to "Todhunter's Euclid" and to "Chauvenet's Geometry."

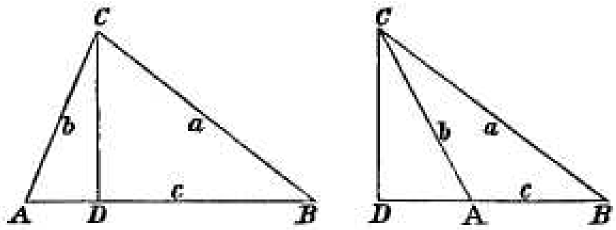
CONTENTS.

CHAPTER XVI.

APPLICATION OF PLANE TRIGONOMETRY TO SOME OF THE PROBLEMS OF MENSURATION.

ART. 165. *To find the area of a triangle, two sides and the included angle being given.*

Suppose the two sides b, c, and the included angle A of the triangle ABC are given; to find its area.



Let the perpendicular CD be drawn to the side c, or c produced.

$CD = b$ sin. A. ((1) Art. 30), (Art. 46).

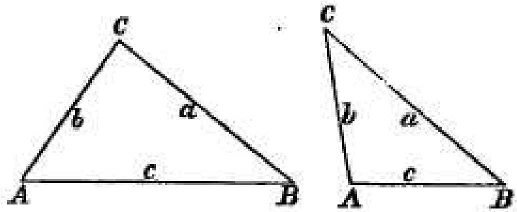
Area $= \frac{1}{2}\, c \times CD$ (Ch. 5, IV.),

$= \frac{1}{2}\, bc$ sin. A.

Therefore, the area of a triangle is equal to one-half the product of any two adjacent sides multiplied by the sine of the included angle.

166. *To find the area of a triangle, of which a side and two adjacent angles are given.*

Suppose c and the angles A and B are given.



$$b = c \times \frac{\sin. B}{\sin. C} = \frac{c \sin. B}{\sin. (A + B)} \text{ (Arts. 48 and 46);}$$

$$\text{Area} = \tfrac{1}{2}\, bc \sin. A = \frac{c^2}{2} \frac{\sin. A \sin. B}{\sin. (A + B)}.$$

167. *To find the area of a triangle, of which the three sides are given.*

a, b, and c being given, it is required to find the area of ABC.

Let $\frac{a+b+c}{2} = s$; then $\frac{b+c-a}{2} = s - a$;

$$\frac{a-b+c}{2} = s - b; \text{ and } \frac{a+b-c}{2} = s - c.$$

Area $= \frac{1}{2}\, bc \sin. A$ (Art. 165).

$= bc \sin. \frac{1}{2} A \cos. \frac{1}{2} A$ ((a) Art. 73).

$$= bc \sqrt{\frac{(s-b)(s-c)}{bc}} \times \sqrt{\frac{s(s-a)}{bc}}$$

((5) and (7) Art. 61).

$$= \sqrt{s(s-a)(s-b)(s-c)}.$$

Therefore, the area of a triangle is equal to the square root of the product of four factors; viz., the half sum of the sides and the three quantities obtained