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W. J. C. MILLER

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MATHEMATICAL QUESTIONS,

WITH THEIR

SOLUTIONS.

FROM THE "EDUCATIONAL TIMES."

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Additional Solutions not published in the "Educational Times."

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(β) Two points may be found, through either of which if any straight line be drawn cutting the given circles, the segments within the ring shall subtend equal angles at the other.
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1488. If two lines be divided homographically, prove by Elementary Geometry that

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If $a < \left(\frac{P+Q}{Q}\right)^2 b$, show that, after Q has left the table, the direction of its motion will at certain instants be vertical, provided $Q < P$; and determine its positions at those instants. 43

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In particular, examine the case in which the three given angles are together equal to two right angles.

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$$\left[(2\rho)^2-(\xi+\gamma)^2\right](\xi^2+\nu^2) = 4\rho^2(\rho^2-\gamma\xi),$$

ξ and ν being the tangential coordinates of the curve, while ρ and γ are the reciprocals of the radius of the given circle, and of the distance of the radiant point from its centre.

When the incident rays are parallel, $\gamma=0$, and the equation becomes $(4\rho^2-\xi^2)\nu^2 = (2\rho^2-\xi^2)^2$. 1

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$$x^2y+xy^2 = 7 \ldots\ldots (1),$$
$$x^7y^3+x^3y^7 = 1608\tfrac{3}{4} \ldots\ldots (2).$$

Also find x, y, z from the equations

$$(x^{16}+y^{16}+z^{16})^3+(x^8+y^8)^2 = 31 \ldots\ldots (1),$$
$$(x^{16}+y^{16}+z^{16})^2(x^8+y^8+z^8)^2 = 729 \ldots\ldots (2),$$
$$(x^8+y^8)^2+(x^8+y^8+z^8)^3 = 31 \ldots\ldots (3).$$ 16

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