AN ELEMENTARY TREATISE ON THE DIFFERENTIAL AND INTEGRAL CALCULUS, FOR THE USE OF COLLEGES AND SCHOOLS

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An elementary treatise on the differential and integral calculus, for the use of colleges and schools by G. W. Hemming

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G. W. HEMMING

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PREFACE TO THE SECOND EDITION.

MANY corrections and additions have been made in the present Edition, chiefly with the view of fitting it for younger students by bringing out the leading principles of the subject with greater clearness. Some additional illustrative examples are given, but for purposes of practice the student is still referred to Gregory's Collection of Examples.

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FIRST BOOK ON PLANE TRIGONOMETRY, comprising Geometrical Trigonometry, and its Application to Surveying, with numerous Examples for the use of Schools.

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