

**AN ELEMENTARY TREATISE ON  
THE DIFFERENTIAL AND  
INTEGRAL CALCULUS, WITH  
EXAMPLES AND APPLICATIONS**

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An Elementary Treatise on the Differential and Integral Calculus, with Examples and Applications by George A. Osborne

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**GEORGE A. OSBORNE**

**AN ELEMENTARY TREATISE ON  
THE DIFFERENTIAL AND  
INTEGRAL CALCULUS, WITH  
EXAMPLES AND APPLICATIONS**



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## PREFACE.

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THIS work, intended as a text-book for colleges and scientific schools, is based on the method of limits, as the most rigorous and most intelligible form of presenting the first principles of the subject. The method of limits has also the important advantage of being a familiar method; for it is now so generally introduced in the study of the more elementary branches of mathematics, that the student may be assumed to be fully conversant with it on beginning the Differential Calculus.

The rules or formulæ for differentiation in Chapter III. differ in one respect from those in similar text-books, in being expressed in terms of  $u$  instead of  $x$ ,  $u$  being any function of  $x$ . They are thus directly applicable to all expressions, without the aid of the usual theorem concerning a function of a function.

After acquiring the processes of differentiation, the student in Chapter V. is introduced to the differential notation, as a convenient abbreviation of the corresponding expressions by differential coefficients. This notation has manifest advantages in the study of the Integral Calculus and in its applications.

In Chapter IX. and subsequent pages I have introduced for Partial Differentiation the notation  $\frac{\partial}{\partial x}$ , which has recently come into such general use.

The chapters on Maxima and Minima have been placed after the applications to curves, as the consideration of that subject is much simplified by representing the function by the ordinate of a curve. Maxima and Minima may be taken, if desired, with equal advantage immediately after Chapter XIII.

In Chapter X., Integral Calculus, I have taken the problem of finding the Moment of Inertia of a plane area, as a better illustration of double integration than that of finding the area itself. The student more readily comprehends the independent variation of  $x$  and  $y$  in the double integral,

$$\iint (x^2 + y^2) dx dy, \text{ than in } \iint dx dy.$$

A few pages of Chapter XII., Integral Calculus, are devoted to a description of the Hyperbolic Functions together with their differentials, and a comparison is made with the corresponding Circular Functions.

G. A. OSBORNE.

# CONTENTS.



## DIFFERENTIAL CALCULUS.

### CHAPTER I.

#### FUNCTIONS.

ARTS.		PAGES.
1-4.	Definition and Classification of Functions .....	1, 2
5.	Notation of Functions. Examples .....	3, 4

### CHAPTER II.

#### DIFFERENTIAL COEFFICIENT.

6, 7.	Limit. Increment .....	5
8-10.	Differential Coefficient. Examples .....	6-9

### CHAPTER III.

#### DIFFERENTIATION.

11-13.	Differentiation of Algebraic Functions. Examples .....	10-21
14-16.	Differentiation of Logarithmic and Exponential Functions. Examples .....	21-27
17, 18.	Differentiation of Trigonometric Functions. Examples ..	27-32
19, 20.	Differentiation of Inverse Trigonometric Functions. Ex- amples .....	32-37
21, 22.	Differentiation of Inverse Function and Function of a Function. Examples .....	37-40

### CHAPTER IV.

#### SUCCESSIVE DIFFERENTIATION.

23, 24.	Definition and Notation .....	41
25.	The $n$ th Differential Coefficient. Examples .....	42-45
26.	Leibnitz's Theorem. Examples .....	45-47

## CHAPTER V.

## DIFFERENTIALS.

ARTS.		PAGES.
27.	Differentials as related to Differential Coefficients .....	48, 49
28.	Differentiation by Differentials .....	49
29.	Successive Differentials. Examples.....	50, 51

## CHAPTER VI.

## IMPLICIT FUNCTIONS.

30.	Differentiation of Implicit Functions. Examples.....	52-54
-----	--	-------

## CHAPTER VII.

## EXPANSION OF FUNCTIONS.

32-36.	Maclaurin's Theorem. Examples .....	55-60
37-41.	Taylor's Theorem. Examples .....	60-63
42-45.	Rigorous Proof of Taylor's Theorem .....	64, 65
46-49.	Remainder in Taylor's and Maclaurin's Theorems .....	66-68

## CHAPTER VIII.

## INDETERMINATE FORMS.

50, 51.	Limiting Value of a Fraction.....	69
52, 53.	Evaluation of $\frac{0}{0}$ . Examples .....	70-73
54-57.	Evaluation of $\frac{\infty}{\infty}$ , $0x$ , $x - \infty$ . Examples .....	73-76
58.	Evaluation of Exponential Forms. Examples.....	76-78

## CHAPTER IX.

## PARTIAL DIFFERENTIATION.

59, 60.	Partial Differential Coefficients of First Order. Examples .....	79, 80
61-63.	Partial Differential Coefficients of Higher Orders. Examples .....	80-82
64, 65.	Total Differential of Functions of Several Variables. Examples .....	82-84
66.	Condition for an Exact Differential. Examples.....	85
67.	Differentiation of Implicit Functions.....	86
68, 69.	Taylor's Theorem for Several Variables.....	87, 88

CONTENTS.

vii

CHAPTER X.

CHANGE OF VARIABLES IN DIFFERENTIAL  
COEFFICIENTS.

ARTS.		PAGES.
70.	Changing from $x$ to $y$ .....	89
71, 72.	Changing from $y$ to $x$ .....	90
73.	Changing from $x$ to $z$ . Examples .....	90-92

CHAPTER XI.

REPRESENTATION OF VARIOUS CURVES.

74-85.	Rectangular Co-ordinates .....	93-98
86-93.	Polar Co-ordinates .....	98-102

CHAPTER XII.

DIRECTION OF CURVE. TANGENT AND NORMAL.  
ASYMPTOTES.

94-97.	Direction of Curve. Subtangent and Subnormal. Examples .....	103-108
98, 98½.	Differential Coefficient of the Arc.....	108, 109
99.	Equation of the Tangent and Normal. Examples ...	109-112
100-106.	Asymptotes. Examples .....	112-116

CHAPTER XIII.

DIRECTION OF CURVATURE. POINTS OF INFLEXION.

107-109.	Direction of Curvature.....	117
110.	Points of Inflexion. Examples.....	118, 119

CHAPTER XIV.

CURVATURE. CIRCLE OF CURVATURE. EVOLUTE  
AND INVOLUTE.

111-113.	Definition of Curvature; Uniform and Variable.....	120, 121
114, 115.	Radius of Curvature. Examples .....	121-124
116.	Centre of Curvature .....	124, 125
117-121.	Evolute and Involute. Examples.....	125-128

## CHAPTER XV.

ARTS.	ORDER OF CONTACT. OSCULATING CIRCLE.	PAGES.
122, 123.	Consecutive Common Points .....	120, 130
124, 125.	Osculating Curves .....	130, 131
126-128.	Analytical Conditions for Contact .....	131-133
129, 130.	Osculating Circle. Examples .....	133-136

## CHAPTER XVI.

## ENVELOPES.

131-133.	Series of Curves. Definition of Envelope .....	137, 138
134-136.	Equation of Envelope .....	138-140
137.	Evolute, the Envelope of Normals. Examples .....	140-144

## CHAPTER XVII.

## SINGULAR POINTS OF CURVES.

138-141.	Multiple Points .....	145-148
142, 143.	Points of Osculation. Cusps .....	149, 150
144.	Conjugate Points. Examples .....	150-152

## CHAPTER XVIII.

MAXIMA AND MINIMA OF FUNCTIONS OF ONE  
INDEPENDENT VARIABLE.

145-149.	Definition. Conditions for Maxima and Minima derived from Curves .....	153-157
150, 151.	Conditions for Maxima and Minima by Taylor's Theorem. Examples .....	157-162
	Problems in Maxima and Minima .....	162-164

## CHAPTER XIX.

MAXIMA AND MINIMA OF FUNCTIONS OF SEVERAL  
INDEPENDENT VARIABLES.

152-155.	Definition. Conditions for Maxima and Minima by Taylor's Theorem. Examples .....	165-171
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