

**NEWTON'S PRINCIPIA,
SECTIONS I, II, III, WITH NOTES
AND ILLUSTRATIONS. ALSO A
COLLECTION OF PROBLEMS**

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Newton's Principia, Sections I, II, III, with Notes and Illustrations. Also a Collection of Problems by Sir Isaac Newton & Percival Frost

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SIR ISAAC NEWTON & PERCIVAL FROST

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NEWTON'S PRINCIPIA.

SECTIONS I. II. III.

WITH

NOTES AND ILLUSTRATIONS.

ALSO

A COLLECTION OF PROBLEMS

PRINCIPALLY INTENDED AS EXAMPLES OF NEWTON'S METHODS.

BY

PERCIVAL FROST, M.A.

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MATHEMATICAL LECTURER OF KING'S COLLEGE.

Principiis enim cognitis, multo facilius extrema intelligetis.—CICERO.

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PREFACE TO THE SECOND EDITION.

IN publishing the following work, my principal intention is to explain difficulties, which may be encountered by the student on first reading the *Principia*, and to illustrate the advantages of a careful study of the methods employed by Newton, by showing the extent to which they may be applied in the solution of problems. I have also endeavoured to give assistance to the student who is engaged in the study of the higher branches of Mathematics, by representing in a geometrical form several of the processes employed in the Differential and Integral Calculus, and in the analytical investigations of Dynamics.

In my version of the first section and the beginning of the second I have adhered as closely as I could to the original form; and, in the cases in which sections have been interpolated, or the form of demonstration changed, I have indicated such changes and interpolations by brackets.

Although it is generally advisable not to deviate from Newton's words in the demonstrations of the Lemmas, yet in many cases, I suppose, purposely, he expressed himself very concisely, as in Lemmas IV. and X.; and he was contented with simply giving the enunciation of Lemma V.; in these cases, therefore, interpolations are made which, I believe, are in accordance with Newton's plan of demonstration.

Throughout the Problems and Theorems which depend upon the sixth proposition, the variations are replaced by equations; by this method of treating the subject, I conceive that clearer ideas of the meaning of each step are obtained by the student.

I take this opportunity to acknowledge the great assistance which I have derived in the preparation of my notes, from the study of Whewell's *Method of Limits*, from which the Articles 55—60 have been almost entirely taken; I have also made use of several editions of Newton, and especially of Carr's.

The Problems are principally selected from the papers set in the examinations for the Mathematical Tripos, and in the course of the College examinations; the results of these problems are given either in the statements or at the end of the work, but I have not thought it advisable to supply hints for the solution, because I imagine that the student would have been deprived thereby of the advantages which it is the object of a problem to secure. It is only necessary to add that I have been careful to introduce no problems which are not capable of solution by methods given in the work.

I desire to express my thanks to Mr Hadley of St John's College for several valuable suggestions, and also to Mr Cockshott of Trinity College, and to Mr King of Jesus College, for their kindness in correcting the errors of the press, and in testing the accuracy of the problems, which, I believe, are nearly free from mistakes.

PERCIVAL FROST.

CAMBRIDGE,

November 13, 1863.

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